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Consider: Stability, power gain, bandwidth, noise, dc bias. Start with a set of specifications and select proper transistor. Next, determine transistor loading (source and load reflection coeff.) for particular stability and gain criteria.

Power gain equations

see fig. 3.2.1 for various powers. Gain definitions:

$$G_T = \frac{P_L}{P_{avs}} = \frac{\text{power delivered to load}}{\text{power available from source}} = \text{transducer} \quad (1)$$

$$G_p = \frac{P_L}{P_{in}} = \frac{\text{power delivered to load}}{\text{power input to network}} = \text{operating} \quad (2)$$

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{\text{power available from network}}{\text{power available from source}} = \text{available} \quad (3)$$

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$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{IN}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (4)$$

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{OUT}\Gamma_L|^2} \quad (5)$$

$$G_P = \frac{1}{1 - |\Gamma_{IN}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = f(\Gamma_L, [S]) \quad (6)$$

$$G_A = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_L|^2} = f(\Gamma_s, [S]) \quad (7)$$

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad \Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \quad (8)$$


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- Source and load mismatch factors:

$$M_s = \frac{P_{IN}}{P_{AVS}} = \frac{(1 - |\Gamma_s|^2)(1 - |\Gamma_{IN}|^2)}{|1 - \Gamma_s \Gamma_{IN}|^2} \left( = \frac{4R_s R_{IN}}{|Z_s + Z_{IN}|^2} \right)$$

$$M_L = \frac{P_L}{P_{AVN}} = \frac{(1 - |\Gamma_L|^2)(1 - |\Gamma_{OUT}|^2)}{|1 - \Gamma_L \Gamma_{OUT}|^2} \left( = \frac{4R_{OUT} R_L}{|Z_{OUT} + Z_L|^2} \right)$$

Use  $M_s$  and  $M_L$  to relate various powers and gains:

$$P_{IN} = P_{AVS} M_s = P_{AVS} |\Gamma_{IN} = \Gamma_s^*|$$

$$P_L = P_{AVN} M_L = P_{AVN} |\Gamma_L = \Gamma_{OUT}^*|$$

$$G_T = \frac{P_L}{P_{AVS}} = \frac{P_L}{P_{AVN}} \frac{P_{IN}}{P_{AVS}} = G_p \frac{P_{IN}}{P_{AVS}} = G_p M_s$$

$$G_A = \frac{P_L}{P_{AVS}} = \frac{P_L}{P_{AVS}} \frac{P_L}{P_{AVN}} = \frac{P_L}{M_L}$$



## Stability

Stability = resistance to “spontaneous” oscillations. Possible if either input or output port has negative resistance, i.e.  $|\Gamma_{IN}| > 1$  or  $|\Gamma_{OUT}| > 1$ . For unilateral device that means  $|S_{11}| > 1$  and  $|S_{22}| > 1$ . See fig. 3.3.1 .

**Unconditionally stable** at  $f$  if real parts of  $Z_{IN}$  and  $Z_{OUT}$  are greater than zero for all passive load and source impedances.

If 2-port is **not unconditionally stable** it is **potentially unstable**, i.e. some passive load and source terminations result in input and output impedances with  $Re(Z) < 0$ .



Conditions for unconditional stability:

$$|\Gamma_s| < 1, \quad |\Gamma_L| < 1 \quad (9)$$

$$|\Gamma_{IN}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \quad (10)$$

$$|\Gamma_{OUT}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{22}\Gamma_s} \right| < 1 \quad (11)$$

From this determine where  $|\Gamma_{IN}| = |\Gamma_{OUT}| = 1$

$$\left| \Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \quad (12)$$



$$\left| \Gamma_s - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \quad (13)$$

with  $\Delta = S_{11}S_{22} - S_{12}S_{21}$  is S-matrix determinant.

Geometrical interpretation: in polar coordinates these are two circles displaced by some amount from origin.

For  $|\Gamma_{TN}| = 1$ ,  $\Gamma_L$  values satisfying the conditions for unconditional stability will be on one side of the circle which has:

$$\text{center: } C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad \text{radius: } r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

This is output stability circle

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Figs. 3.3.2 and 3.3.3. Which side of the circle is unconditionally stable (and which one is not)?

Observe: for  $\Gamma_L = 0, Z_L = Z_0 \Rightarrow |\Gamma_{IN}| = |S_{11}|$ . For  $|S_{11}| < 1 \Rightarrow |\Gamma_{IN}| < 1$  when  $Z_L = Z_0$  (origin).

$\Rightarrow$  Based on value of  $S_{11}$ , origin is either in stable region or not. For  $|S_{11}| < 1$  it is part of the stable region. Conversely, if  $|S_{11}| > 1$  then origin is part of the potentially unstable region. (fig. 3.3.3).

Note that  $|\Gamma_L| < 1$  is required in the above.

Similar analysis for input stability circle (fig. 3.3.4).

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For unconditional stability circles do not cross S-chart  $\Rightarrow$  for no value of  $Z_L$  or  $Z_s$  will the circuit oscillate. for  $|S_{11}| < 1$  we have  $\|C_L - r_L\| > 1$  and for  $|S_{22}| < 1$  we have  $\|C_s - r_s\| > 1$ .

Note that  $|S_{11}| > 1$  and  $|S_{22}| > 1$  are automatically excluded from unconditional stability since  $\Gamma_s = 0$  produces  $|\Gamma_{OUT}| > 1$  and  $\Gamma_L = 0$  produces  $|\Gamma_{IN}| > 1$

Different form for the conditions (for unconditional stability)

Define stability factor K

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1 \quad (14)$$

$$1 - |S_{11}|^2 > |S_{12}S_{21}| \quad 1 - |S_{22}|^2 > |S_{12}S_{21}| \quad (15)$$


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written differently:

$$K > 1 \text{ and } B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0 \quad (16)$$

or even simpler

$$K > 1 \text{ and } |\Delta| < 1 \quad (\Delta = S_{11}S_{22} - S_{12}S_{21}) \quad (17)$$

Ex. on p. 223. :  $K > 1$  but  $|\Delta| > 1 \Rightarrow$  potentially unstable.

Unilateral amplifier:  $K \rightarrow \infty$  and for unconditionally stability we only need  $|S_{11}| < 1$  and  $|S_{22}| < 1$ .

It is always possible to make the circuit stable if the total input and output loop resistances are positive (fig. 3.3.1)

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$$\operatorname{Re}(Z_s + Z_{IN}) > 0 \quad \text{and} \quad \operatorname{Re}(Z_L + Z_{OUT}) > 0 \quad (18)$$

Accomplished by:

- resistively loading the transistor
- adding negative feedback

Not recommended for narrowband design.

Example 3.3.2.

By adding resistances we change  $\Gamma_s$  and  $\Gamma_L$ , but loose power too.  
Added resistance can be absorbed into transistor S-matrix (how?)

Negative feedback could be used to make  $S_{12} = 0$  to stabilize transistor; not commonly done.

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Unilateral network:  $S_{12} = 0 \Rightarrow \Gamma_{IN} = S_{11}$ ,  $\Gamma_{OUT} = S_{22}$ . In that case: Unilateral transducer power gain:

$$G_{TU} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (19)$$

Think of it as a product of three components:  $G_{TU} = G_s \cdot G_0 \cdot G_L$ . see fig. 3.2.2.

$G_s$  affects the degree of mismatch between  $\Gamma_s$  and  $S_{11}$ . It is passive, but can have gain  $> 1$ . Since there is an “intrinsic” mismatch loss between  $Z_0$  and  $S_{11}$  (i.e. between  $\Gamma$  and  $S_{11}$ )  $\Rightarrow$  decreasing that loss provides gain. Same for  $G_L$ .

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For dB-s:  $G_{TU}(dB) = G_s(dB) + G_0(dB) + G_L(dB)$ .

Optimize (match)  $\Gamma_s$  and  $\Gamma_L$  for max.  $G_s$  and  $G_L \Rightarrow \Gamma_s = S_{11}^*$  and  $\Gamma_L = S_{22}^*$  ( $|S_{11}| < 1, |S_{22}| < 1$ ).

$$\Rightarrow G_{s,max} = \frac{1}{1 - |S_{11}|^2} \quad G_{L,max} = \frac{1}{1 - |S_{22}|^2} \quad (20)$$

$$\Rightarrow G_{TU,max} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2} \quad (21)$$



Constant gain circles: unilateral case

$$G_{TU} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (22)$$

or  $G_{TU} = G_S \cdot G_0 \cdot G_L$ . For general analysis write  $G_S$ ,  $G_L$  as

$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii}\Gamma_i|^2} \quad (23)$$

Two cases:

1. unconditionally stable:  $|S_{ii}| < 1$
  2. potentially unstable:  $|S_{ii}| > 1$
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Case 1. Max. value of  $G_i \Rightarrow$  match  $\Gamma_i$  and  $S_{ii}$ , ie.  $\Gamma_i = S_{ii}^*$

$$\Rightarrow G_{ii} = \frac{1}{1 - |S_{ii}|^2} \quad (24)$$

Terminations that produce  $G_{i,max}$  are called optimum terminations.

From definition of  $G_i$ , if  $|\Gamma_i| = 1 \Rightarrow G_i = 0$ . Other values of  $|\Gamma_i|$  produce values  $0 \leq G_i \leq G_{i,max}$ .

Analysis of  $G_i$  via S-chart leads to constant gain circles.

Introduce normalized gain factor:

$$g_i = \frac{G_i}{G_{i,max}} = G_i(1 - |S_{ii}|^2), \quad \text{and} \quad 0 \leq g_i \leq 1 \quad (25)$$



Also define  $\Gamma_i = U_i + jV_i$  and  $S_{ii} = A_{ii} + jB_{ii}$  and plug it into eq. 25. Many manipulations later...

$$\left[ U_i - \frac{g_i A_{ii}}{1 - |S_{ii}|^2 (1 - g_i)} \right]^2 + \left[ V_i - \frac{g_i B_{ii}}{1 - |S_{ii}|^2 (1 - g_i)} \right] = \left[ \frac{\sqrt{1 - g_i} (1 - |S_{ii}|^2)}{1 - |S_{ii}|^2 (1 - g_i)} \right]^2$$

These are circles with  $g_i$  as parameter. Centers are at:

$$U_c = \frac{g_i A_{ii}}{1 - |S_{ii}|^2 (1 - g_i)}, \quad V_c = -\frac{g_i B_{ii}}{1 - |S_{ii}|^2 (1 - g_i)} \quad (26)$$

with radius  $R_i = \frac{\sqrt{1 - g_i} (1 - |S_{ii}|^2)}{1 - |S_{ii}|^2 (1 - g_i)}$  (27)

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Writing differently, distance from origin

$$d_i = \sqrt{U_c^2 + V_c^2} = \frac{g_i |S_{ii}|}{1 - |S_{ii}|^2 (1 - g_i)} \quad (28)$$

and angle

$$\tan \alpha = \frac{V_c}{U_c} \Rightarrow \alpha_i = \tan^{-1} \frac{-B_{ii}}{A_{ii}} \quad (29)$$

Const. gain circles located at distance  $d_i$  along the line drawn from origin to point  $S_{ii}^*$  ( $= A_{ii} - jB_{ii}$ ).





Procedure for const. gain circle(s) construction:

- Locate  $S_{11}^*$  and draw line from origin to it. At  $S_{ii}^*$  gain is  $G_{i,max} = 1/(1 - |S_{ii}|^2)$ , and the radius is  $R_{i,max} = 0$  (point).
- find values for  $0 \leq G_i \leq G_{i,max}$  (using  $G_i = (1 - |\Gamma_i|^2)/|1 - S_{ii}\Gamma_i|^2$ ), and calculate  $g_i = G_i/G_{i,max}$ .
- determine  $d_i$ ,  $R_i$  for each  $g_i$  (eqs. 28 and 27 )
- 0 dB circle ( $G_i = 1$ ) always goes through origin, or for  $G_i = 1 \Rightarrow \Gamma_i = 0$  so that

$$g_{i,0dB} = 1 - |S_{ii}|^2 \quad \text{and} \quad R_{i,0dB} = d_{i,0dB} = \frac{|S_{ii}|}{1 + |S_{ii}|^2} \quad (30)$$



Example 3.4.1 shows calculation of optimum terminations, various gains, construction of const. gain circles and design based on those circles.

- determine impedances for optimum terminations
  - find  $G_{s,max} = 1/(1 - |S_{11}|^2) = 2.141$  (= 3.31 dB)
  - find  $G_{L,max} = 1/(1 - |S_{22}|^2) = 1.046$  (= 0.19 dB)
  - find  $G_0 = |S_{21}|^2 = 19.8$  (= 12.97 dB); for total sum them up.
  - construct table of  $g_s, d_s, R_s$  for various  $G_i$ -s (fig. 3.4.4) (why no matching for output?)
  - construct matching network for given gain (fig. 3.4.5)
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### Potentially unstable case (case 2)

Unilateral network:  $|S_{ii}| > 1$  (i.e. negative input resistance is present). Look at expr. 23; for  $\Gamma_i \rightarrow 1/S_{ii} \Rightarrow G_i \rightarrow \infty$ . The critical value is  $\Gamma_{i,c} = 1/S_{ii}$ .

Under “normal” circumstances ( $|S_{11}| < 1$ ) above condition cannot be satisfied by passive terminations.

For  $\Gamma_{i,c} = 1/S_{ii}$  real parts of impedances associated with  $\Gamma_{i,c}$  and  $S_{ii}$  cancel  $\Rightarrow$  no resistance in the circuit!! Oscillations follow.

Remember that for negative resistances we plot the inverse, complex conjugate of the original value; here we plot  $1/S_{ii}^*$  on S-chart and read resistance circles as negative and reactances as given.

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Normalized gain  $g_i = G_i(1 - |S_{ii}|^2)$  can be negative! However, the same formulas for  $R_i$  and  $d_i$  hold (see eqs. 27, 28), except the centers are located along the origin -  $1/S_{ii}$  line.

Need to determine stable region: use conditions:

$$\operatorname{Re}(Z_s) > |\operatorname{Re}(Z_{in})| \quad \text{and} \quad \operatorname{Re}(Z_L) > |\operatorname{Re}(Z_{out})| \quad (31)$$

i.e., the total resistance in the circuit is still positive.

Example 3.4.2 (fig. 3.4.6).

- locate  $1/S_{11}^*$  to find  $Z_{in}$
  - find  $1/S_{11}$  for line on which centers of const. gain circles will be
- 



- Stable region is where total resistance is positive, i.e. inside the shaded const. resistance circle
- optimum output termination from  $\Gamma_L = S_{22}^* = 0.6\angle 80^\circ$ . After finding  $\Gamma_L$  on S-chart, read off the  $Z_L$  value.
- $G_s = 5\text{dB} \Rightarrow g_s = 3.16 [1 - (2.27)^2] = -13.123$
- plug  $g_s$  into formulas for  $R_s$  and  $d_s$ :  $R_s = 0.217$ ,  $d_s = 0.415$
- for largest stability pick up  $\Gamma_s$  that has largest  $\text{Re}(\Gamma_s)$  on any given const. gain circle (pt. A).



## Unilateral figure of merit

When is it permissible to take  $S_{12} = 0$  (unilateral device)?

$$\frac{G_T}{G_{TU}} = \frac{1}{|1 - X|^2}, \quad X = \frac{S_{12}S_{21}\Gamma_s\Gamma_L}{(1 - S_{11}\Gamma_s)(1 - S_{22}\Gamma_L)} \quad (32)$$

$$\text{bounded by } \frac{1}{(1 + |X|)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1 - |X|)^2} \quad (33)$$

$G_{TU}$  max when conj. matching provided:  $\Gamma_s = S_{11}^*$  etc. giving max error introduced by  $S_{12} = 0$ .

$$\frac{1}{(1 + U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1 - U)^2} \Rightarrow \quad (34)$$

$$\text{unilateral figure of merit } U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} \quad (35)$$


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### Simultaneous conjugate match: Bilateral case

What to do if  $S_{12} \neq 0$  (or,  $U$  is not small)?

Most general expressions for  $\Gamma_{IN}$  and  $\Gamma_{OUT}$  still hold:

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S'_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad (= \Gamma_s^*) \quad (36)$$

$$\Gamma_{OUT} = S'_{22} + \frac{S'_{12}S'_{21}\Gamma_s}{1 - S'_{11}\Gamma_s} \quad (= \Gamma_L^*) \quad (37)$$

For max. power transfer  $\Gamma_{IN} = \Gamma_s^*$  and  $\Gamma_{OUT} = \Gamma_L^*$ . fig. to illustrate.

Solve two eqs. with two unknowns:  $\Gamma_L$ ,  $\Gamma_s$ . Call the solution  $\Gamma_{Ms}$  and  $\Gamma_{ML}$  (for simultaneously matched case).

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Solution:

$$\Gamma_{Ms} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \quad \Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \quad (38)$$

$$C_1 = S_{11} - \Delta S_{22}^* \quad B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad (39)$$

$$C_2 = S_{22} - \Delta S_{11}^* \quad B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \quad (40)$$

Which sign to use above? First,  $|\Gamma_{Ms}|$ ,  $\Gamma_{ML} < 1$  is necessary for unconditional stability.

Further analysis shows the necessary condition to be  $K > 1$  where  $K$  is stability factor! From before we know the additional requirement:  $|\Delta| < 1 \Rightarrow B_1 > 0$  and  $B_2 > 0$

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All of this  $\Rightarrow$  negative signs (-) should be used in  $\Gamma_{M_s}$ ,  $\Gamma_{M_L}$  when calculating the simultaneous con. match for unconditionally stable two-port network.

Potentially unstable case: analysis done in terms of  $G_P$ ,  $G_A$  (but see recent literature).

What is the max.  $G_T$  associated with  $\Gamma_{M_s}$ ,  $\Gamma_{M_L}$ ?

$$\begin{aligned}
 G_{T,max} &= \frac{(1 - |\Gamma_{M_s}|^2)|S_{21}|^2(1 - |\Gamma_{M_L}|^2)}{|(1 - S_{11}\Gamma_{M_s})(1 - S_{22}\Gamma_{M_L}) - S_{12}S_{21}\Gamma_{M_s}\Gamma_{M_L}|^2} \\
 &= \frac{1}{1 - |\Gamma_{M_s}|^2} |S_{21}|^2 \frac{1 - |\Gamma_{M_L}|^2}{|1 - S_{22}\Gamma_{M_L}|^2} \quad (41)
 \end{aligned}$$

(see eq. 3.2.1 — no  $\Gamma_{IN}$  or  $\Gamma_{OUT}$  present there).

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Further manipulation:

$$G_{T,max} = \frac{|S_{21}|}{|S_{12}|} (K - \sqrt{K^2 - 1}) \quad (42)$$

note:  $G_{T,max}$  depends only on transistor parameters (S-matrix).  $\Gamma_{Ms}$ ,  $\Gamma_{ML}$  don't enter the picture because they are determined by conjugate matching requirement.

If we had some control of [S], what would max.  $G_{T,max}$  be? Since  $K - \sqrt{K^2 - 1}$  falls monotonically to 0, max value is for  $K = 1$ . The value of  $G_{T,max}$  for  $K=1$  is called maximum stable gain (MSG),  $G_{MSG} = |S_{21}|/|S_{12}|$ . Condition  $K=1$  can be achieved by resistive loading or by feedback

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Constant gain circles: bilateral case

Conditions:  $K > 1$  and  $|\Delta| < 1$ .

Design may call for  $G_T \neq G_{T,max}$  (produced by conjugate matching); what to do then? Need to choose  $\Gamma_s, \Gamma_L$  differently. Start with:

$$\begin{aligned}
 G_T &= \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{IN}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \\
 &= \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{OUT}\Gamma_L|^2} = G'_s G_0 G_L \quad (43)
 \end{aligned}$$

$$G'_s = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{IN}\Gamma_s|^2} \quad G_0 = |S_{21}|^2 \quad G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (44)$$



Compare this with expressions in unilateral case

$$G_s = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} \quad G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (45)$$

$\Rightarrow \Gamma_{IN}$  replaced  $S_{11}$

General procedure follows unilateral case, but it is iterative because  $\Gamma_{IN}$  depends on choice of  $\Gamma_L$ .

Procedure:

- Pick  $\Gamma_L$  for given  $G_L$  gain (see eq. above). Const. gain circles can be drawn using unilateral gain formulas

$$g_L = G_L / G_{L,max} = G_L (1 - |S_{22}|^2) \quad (46)$$


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$$R_L = \frac{\sqrt{1 - g_L}(1 - |S'_{22}|^2)}{1 - |S'_{22}|^2(1 - g_L)} \quad (47)$$

$$d_L = \frac{g_L |S'_{22}|}{1 - |S'_{22}|^2(1 - g_L)} \quad (48)$$

angle determined by  $S'_{22}$ : line connecting the origin with  $S'_{22}$  will have centers of circles on it.

- Calculate  $\Gamma_{IN}$  from  $\Gamma_{IN} = S_{11} + S_{12}S_{21}\Gamma_L/(1 - S_{22}\Gamma_L)$
- $G'_s$  const. gain circles drawn using the same eqs. ( 46, 47, 48) but  $\Gamma_{IN}$  goes where  $S_{11}$  would normally go:

$$g_s = G'_s / G_{s,max} = G'_s(1 - |\Gamma_{IN}|^2) \quad (49)$$


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- $$R_s = \frac{\sqrt{1 - g_s}(1 - |\Gamma_{IN}|^2)}{1 - |\Gamma_{IN}|^2(1 - g_s)} \quad (50)$$

$$d_s = \frac{g_s |\Gamma_{IN}|}{1 - |\Gamma_{IN}|^2(1 - g_s)} \quad (51)$$

angle determined by  $\Gamma_{IN}^*$  (instead of  $S_{11}^*$ )

Select the desired  $\Gamma_s$  for a given  $G'_s$  gain. The available values of  $G'_s$  may not be satisfactory, e.g.  $G_{s,max} = 1/(1 - |\Gamma_{IN}|^2)$  may be too small, or values of possible  $\Gamma_s$  may not be good etc  $\Rightarrow$  go back to start and choose different  $\Gamma_L$  and repeat the procedure.

- Once  $\Gamma_s$  and  $\Gamma_L$  are picked up, design matching networks as before.

Example 3.6.1. Operate at 6 GHz with  $G_{T,max}$ . Fig. 3.6.3.

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- calc.  $K$  and  $|\Delta|$  for stability:  $K = 1.504$ ,  $\Delta = 0.301 \angle 109^\circ$  (is it unconditionally stable?)
- is FET unilateral? calculate  $U = 0.1085$  so that

$$-0.89 \text{ dB} \leq \frac{G_T}{G_{TU}} < 1 \text{ dB} \Rightarrow S_{12} \neq 0 \quad (52)$$

- calc. simultaneous conjugate match (need  $B_1, B_2, C_1, C_2$ )  $\Rightarrow \Gamma_{M_s} = 0.76 \angle 177^\circ$ ,  $\Gamma_{M_L} = 0.71 \angle 103^\circ$
  - from  $K, S_{21}, S_{12} \Rightarrow G_{T,max} = 11.3 \text{ dB}$ .
  - given  $\Gamma_{M_s}, \Gamma_{M_L}$  find matching networks. From S-chart:  $Y_{M_s} = 7.2 - j1.23 = 0.144 - j0.0246 \text{ S}$ , and  $Y_{M_L} = 0.414 - j1.19$ ,  $= 0.0083 - j0.0238 \text{ S}$ . See fig. 3.6.3 and 3.6.4 for design.
- 



- calc. real lengths for given material (say, Duroid)
- check other frequencies to make sure that chosen  $\Gamma$ -s provide stable operation.

How about potentially unstable bilateral case ( $K < 1$  or  $|\Delta| > 1$ )?

Turns out that design procedure based on  $G_T$  is not good and is better done based on operating power gain equation.

Conclusion on bilateral case: anything but design for  $G_{T,max}$  is tedious (and better alternative exists); if transistor is unconditionally stable simultaneous conjugate match can be found resulting in  $G_{T,max}$ .

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