Operating and available power gain circles

Operating power gain

$$G_P = \frac{P_L}{P_{IN}} = \frac{\text{power delivered to load}}{\text{power input to network}} \quad (1)$$

and it is independent of source impedance.

Design procedure is simple for both unconditionally stable and potentially unstable transistors.

Unconditionally stable bilateral case:

$$G_P = \frac{1}{1 - |\Gamma_{IN}|} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (2)$$

$$G_P = |S_{21}|^2 \frac{(1 - |\Gamma_L|^2)}{\left(1 - \left|\frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}\right|^2\right)|1 - S_{22}\Gamma_L|^2} = |S_{21}|^2 g_p \tag{3}$$

$$g_p = \frac{1 - |\Gamma_L|^2}{1 - |S_{11}|^2 + |\Gamma_L|^2(|S_{22}|^2 - |\Delta|^2) - 2\mathrm{Re}(\Gamma_L C_2)} \tag{4}$$

$$C_2 = S_{22} - \Delta S_{11}^* \tag{5}$$

G_P, g_p functions only of $[S], \Gamma_L$. Procedure for obtaining const. gain circles as before. Final result:

- Circle radius:

$$R_p = \frac{\left[1 - 2K|S_{12}S_{21}|g_p + |S_{12}S_{21}|^2 g_p^2\right]^{1/2}}{|1 + g_p(|S_{22}|^2 - |\Delta|^2)|} \tag{6}$$

- distance from origin:

$$d_p = \frac{g_p|C_2^*|}{|1 + g_p(|S_{22}|^2 - |\Delta|^2)|} \tag{7}$$

- Angle: g_p is a real number. Looking at expr. for center of the circle (eqs. not given), the direction (angle) of the centers will be determined by C_2^*.

$$C_p = \frac{g_p C_2^*}{1 + g_p(|S_{22}|^2 - |\Delta|^2)} \tag{8}$$

As our intuition tells us, g_p will have its max. for $R_p = 0 \Rightarrow$

$$g_{p,max}^2|S_{12}S_{21}|^2 - 2K|S_{12}S_{21}|g_{p,max} + 1 = 0 \tag{9}$$

$$g_{p,max} = \frac{1}{|S_{12}S_{21}}|(K - \sqrt{K^2 - 1}) \tag{10}$$

$$\Rightarrow G_{P,max} = \frac{|S_{21}|}{|S_{12}|}(K - \sqrt{K^2 - 1}) \tag{11}$$

Given G_P we select Γ_L from constant operating power gain circles. In order to get max. G_P, Γ_L is selected at distance where $g_{p,max} = G_{P,max}/|S_{21}|^2$, i.e. calculate $G_{P,max}$ from eq. 11, find $g_{p,max} = G_{P,max}/|S_{21}|^2$, and plug it into eq. for d_p (eq. 7). Intersection of that with direction of C_2^* gives Γ_L for $G_{P,max}$.

Max. output power obtained if input is conj. matched, i.e. $\Gamma_s = \Gamma_{IN}^*$; in this case the input power is equal to max. available power from source $\Rightarrow G_{T,max} = G_{P,max}$. Note that values of Γ_s, Γ_L that give $G_{P,max}$ are identical to Γ_{Ms}, Γ_{ML}.

Procedure

- Specify G_P; radius, center and angle are given in eqs. above.
- Select the desired Γ_L
- For given Γ_L max. output power is obtained by conjugate matching on input, i.e. with

$$\Gamma_s = \Gamma_{IN}^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \tag{12}$$

This value of Γ_s produces the transducer gain $G_T = G_P$ (if $G_P = G_{P,max}$ is specified at the beginning, G_T will be equal to $G_{P,max}$.)

Example 3.8.1. Design for $G_p = 9$dB.

- $|S_{21}|^2 = 4.235$ or 6.27 dB. $g_p = G_p/|S_{21}|^2 = 1.875$. K, Δ, C_2 calculated before (ex. 3.7.1)
- calculate $R_p = 0.431$ and $C_p = 0.508\angle 103.9°$.
- select some Γ_L point, say pt. A where $\Gamma_L = 0.36\angle 47.5°$ (note the resistance!)
- calculate Γ_s from known $\Gamma_L \Rightarrow \Gamma_s = 0.629\angle 175°$

Same procedure can be used to find $G_{p,max}$ (for simultaneous conjugate matching) $\Rightarrow$ different circle (actually point): $g_{p,max} = 3.24$, $R_{p,max} = 0$, $C_{p,max} = 0.718\angle 103.9°$.

This leads to values for $\Gamma_{L,max} = 0.718\angle 103.9°$. Use it to find $\Gamma_{s,max} = 0.762\angle 177° \Rightarrow$ same as Γ_{Ms}, Γ_{ML} from before.

Constant available power gain circles

$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{\text{P avail. from network}}{\text{P avail. from source}} \tag{13}$$

$$G_A = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1}{1-|\Gamma_{OUT}|^2} \tag{14}$$

Derivation analogous to operating power gain case: exchange $1 \rightarrow 2$, $2 \rightarrow 1$, $L \rightarrow S$.

$$g_a = \frac{G_A}{|S_{21}|^2} \quad C_1 = S_{11} - \Delta S_{22}^* \tag{15}$$

$$R_a = \frac{\left[1 - 2K|S_{12}S_{21}|g_a + |S_{12}S_{21}|^2 g_a^2\right]^{1/2}}{|1 + g_a(|S_{11}|^2 - |\Delta|^2)|} \tag{16}$$

$$C_a = \frac{g_a C_1^*}{1 + g_p(|S_{22}|^2 - |\Delta|^2)} \tag{17}$$

By plotting const. available gain circles a Γ_s can be picked up to produce desired G_A. With Γ_s known, max. power on output is obtained for conjugate matched load so that

$$\Gamma_L = \Gamma_{OUT}^* = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}\right)^* \tag{18}$$

For $\Gamma_L = \Gamma_{OUT}^* \Rightarrow G_T = G_A$.

Potentially unstable bilateral case

We have to worry about stability! Procedure is still the same:

- specify G_p and draw const. operating gain circles. (calc. R_p, C_p)
- draw output stability circle (calc. r_L, C_L; remember that they give Γ_L-s for which $|\Gamma_{IN}| = 1$)
- choose Γ_L in stable region (stay away from circle)
- draw input stability circles (calc. r_s, C_s).
- calculate Γ_{IN} and see if $\Gamma_s = \Gamma_{IN}^*$ is in the stable region
- if Γ_s is not in stable region, or is too close to stability circle: a) select Γ_s arbitrarily, b) select new G_p, c) try different Γ_L

Γ_s, Γ_L should be away from the stability circles to avoid oscillations during tuning or due to variations of parameters.

Example 3.8.2. Design for 10 dB.

- stability: $K = 0.4 < 1 \Rightarrow$ potentially unstable. $G_{MSG} = 31$ $=$ 14.9 dB.
- calc. const. gain circles: $R_p = 0.473, C_p = 0.57\angle 97^\circ$
- output stability circle: $r_L = 0.34, C_L = 1.18\angle 97^\circ$; note that C_L and C_p are on the same line; obvious from their eqs.
- stability region: since $|S_{11}| < 1$, stable region includes origin.

- select Γ_L somewhere on const. gain circle. (different choices possible: max. resistance, min. SWR etc). pt A has $\Gamma_L = 01.\angle 97^\circ$
- on input: $\Gamma_s = \Gamma^*_{OUT} = 0.52\angle 179^\circ$; is it stable?
- input stability circle: $r_s = 1, C_s = 1.67\angle 171^\circ$. Since $|S_{22}| < 1$, origin is in stable region $\Rightarrow \Gamma_s$ is stable point. (construct these on S-chart).

Yet another method for amplifier design

M.L. Edwards et al., "A Deterministic Approach for Designing Conditionally Stable Amplifiers," *IEEE MTT*, vol. 43, no. 7, pp. 1567–1575, 1995.

Basic problem: input and output treated separately even though each affects the other. Fig. 1 for basic procedure for the methods discussed so far:

1. Pick a "suitable" source reflection coefficient Γ_s
2. design input matching network
3. calculate Γ_{OUT} and take $\Gamma_L = \Gamma_{OUT}^*$
4. is Γ_L in stable region?

What to do if Γ_L is not stable?

1. accept the give Γ_L and design the OMN
2. change Γ_L to be stable but "close" to Γ^*_{OUT}
3. change Γ_s and hope that new Γ_L will be stable

Problems: possibly unstable (1) and iterative (2,3).

Distinction: stability on input and output separated. $\Rightarrow$ circuit can be output stable ($|\Gamma_{OUT}| < 1$) but input unstable ($|\Gamma_{IN}| > 1$) and vice-versa.

Definition: **Jointly stable** I/O circuits are those that have $|\Gamma_{OUT}(\Gamma_s)| < 1$ **and** $|\Gamma_{IN}(\Gamma^*_{OUT}(\Gamma_s))| < 1$.

Idea: If stable/unstable region of Γ_L plane could be represented in the Γ_s plane, than we could devise a non-iterative scheme for picking Γ_s and Γ_L. (note: Γ_L is really function of Γ_s since we assume $\Gamma_s = \Gamma^*_{OUT}$). Recall:

$$\Gamma_{IN} = f(\Gamma_L) = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} \tag{19}$$

$$\Gamma_{OUT} = g(\Gamma_s) = \frac{S_{22} - \Delta\Gamma_s}{1 - S_{11}\Gamma_s} \tag{20}$$

Similar eqs. can be obtained for Γ_L (from (19)), and Γ_s (from (20)).

Define:

1. *Output stable region in the source plane* where $|\Gamma_{OUT}(\Gamma_s)| < 1$ ($=$ input stability circle), and
2. *input stable region in the source plane* where $\Gamma_{IN}(|\Gamma^*_{OUT}(\Gamma_s))| < 1$ ($=$ output stability circle, but in Γ_s plane).

For (1), see fig. 2 (two cases: $D_1 > 0$ and $D_1 < 0$, where D_1 depends only on S-matrix).

Section III of the paper shows that $|k| < 1$ *if and only if* stability circle intersects the unit circle. Why is that important?

For $|k| < 1$, stability circle intersects the unit circle in two points that are **common to all available gain circles** which are called invariant points. $\Rightarrow$ geometric relationship between the stability circle, the gain circle and the unit circle can be determined solely by comparing the centers of the circles.

The invariant (intersection) points are given by

$$\Gamma_s^{\pm} = \frac{B_1 \pm \sqrt{4|C_1|^2 - B_1^2}}{2C_1} \tag{21}$$

But, what can we learn about the gain g_a $(G_A = g_a|S_{21}|^2|)$ from all this?

Because of the discussion above, only the position of the center (and they are all on one line) matters. How does g_a change along this line $\Gamma_s = x\hat{c}_s$? Based on the derivative and some reasoning $\Rightarrow$ the gain function g_a is a **monotonic** function of x whenever $|k| < 1$. See fig. 3 for illustration. For $|\Gamma_s| = 1 \Rightarrow g_a = 0$, and $g_a \to \infty$ when Γ_s is on stability circle.

Not very useful, unless we can show input stable region in the source plane.

Through equations (section V) it is shown that the center of this circle is on the same line as available gains and that it intersects the invariant points! $\Rightarrow$ **input stable boundary in the source plane is an available gain circle**.

The gain that corresponds to this boundary (IS) is :

$$g_{is} = \frac{2k}{|S_{12}S_{21}|} \Rightarrow G_{IS} = 2k\left|\frac{S_{21}}{S_{12}}\right| = 2k \text{ MSG} \tag{22}$$

Question: when/if is this g_{is} upper bound for the available gain?

There is a max. of available gain when source impedance Γ_s is required to be passive and **jointly stable**, called max. jointly stable available gain.

The analysis requires examination of different combination of these circles: see Table 1. Two obvious requirements: $|\Gamma_s| < 1$ (passive source region) and $|\Gamma_{OUT}(\Gamma_s)| < 1$ (output stable region). For $|\Gamma_{IN}(\Gamma^*_{OUT}(\Gamma_s))| < 1$ (input stable region) stable region can be either inside or outside the g_{IS} circle.

Different cases in Figs. 4 – 8. Case I in some detail.

Conclusion: when $0 < k < 1$ then the jointly stable region overlaps with the USC, i.e. Γ_s from the overlapping region result in an output whose conjugate match is located on the stable side of the load stability circle. Also, the gain is limited to G_{IS}.

For $-1 < k < 0$ no passive source impedance will result in joint stability.

For $0 < k < 1 \Rightarrow$ the maximum available gain for jointly stable source impedances is $G_{IS} = 2kMSG (= G_{MSM})$.

Similar reasoning can be extended to operating power gain (source exchanged with load).

Example of design for 6 GHz amplifier with conditionally stable transistor.