## Microstrip matching networks

Ease of fabrication: printed circuit boards! (fig. 2.5.1)

Substrates: duroid, quartz, alumina, silicon.

Propagation only quasi-TEM since not all E-M field lines are not entirely in substrate; problem at higher freq.

Phase velocity for quasi-TEM: $v_p = c/\sqrt{\epsilon_{ff}}$. c is speed of light, $\epsilon_{eff} =$ effective relative dielectric constant of the substrate. (**not** $=$ to $\epsilon_r$)

Characteristic impedance: $Z_0 = 1/(v_p C)$, with C $=$ capacitance per unit length of the microstrip.

Wavelength: $\lambda = v_p/f = c/(f\sqrt{\epsilon_{ff}}) = \lambda_0/\sqrt{\epsilon_{ff}}$

How to find $\epsilon_{eff}$ and $Z_0$ ? Preferably in analytical form.

Assuming zero (negligible) thickness of the strip conductor ($t/h < 0.005$):

- For $W/h \leq 1$:

$$Z_0 = \frac{60}{\sqrt{\epsilon_{ff}}} \ln \left( 8\frac{h}{W} + 0.25\frac{W}{h} \right] \tag{1}$$

where

$$\epsilon_{ff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ \left( 1 + 12\frac{h}{W} \right)^{-1/2} + 0.04 \left( 1 - \frac{W}{h} \right)^2 \right] \tag{2}$$

- For $W/h \geq 1$:

$$Z_0 = \frac{120\pi/\sqrt{\epsilon_{ff}}}{W/h + 1.393 + 0.667\ln\left(W/h + 1.444\right)} \tag{3}$$

where

$$\epsilon_{ff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left(1 + 12\frac{h}{W}\right)^{-1/2} \tag{4}$$

- For $W/h \geq 0.6$:

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}\left[\frac{\epsilon_r}{1 + 0.63\left(\epsilon_r - 1\right)\left(W/h\right)^{0.1255}}\right]^{1/2} \tag{5}$$

- For $W/h < 0.6$:

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}\left[\frac{\epsilon_r}{1 + 0.6\left(\epsilon_r - 1\right)\left(W/h\right)^{0.0297}}\right]^{1/2} \tag{6}$$

For design: eqs. relating $Z_0$ with $\epsilon_r$ and $W/h$:

- for $W/h \leq 2$:

$$\frac{W}{h} = \frac{8e^{A}}{e^{2A} - 2} \tag{7}$$

- for $W/h \geq 2$:

$$\frac{W}{h} = \frac{2}{\pi}\left\{B - 1 - \ln(2B - 1) + \frac{\epsilon_r - 1}{2\epsilon_r}\left[\ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r}\right]\right\} \tag{8}$$

where

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right) \tag{9}$$

and $B = 377\pi/(2Z_0\sqrt{\epsilon_r})$.

Non-negligible thickness of conductor $\Rightarrow$ included via increased capacitance $\Rightarrow$ replace strip width W with $W_{eff}$, e.g.

$$\frac{W_{eff}}{h} = \frac{W}{h} + \frac{t}{\pi h}\left(1 + \ln\frac{2h}{t}\right) \tag{10}$$

for $W/h \geq 1/2\pi$.

Example 2.5.1; see table 2.5.4. Use CAD tools (MDS)!

High frequencies: quasi-TEM not valid and $\epsilon_{ff}$ and $Z_0$ are functions of freq. $v_p$ decreases with freq. $\Rightarrow \epsilon_{ff}$ increases. Also $Z_0$ increases with freq. while $W_{eff}$ decreases.

Dispersion neglected below

$$f_0(\text{GHz}) = 0.3\sqrt{\frac{Z_0}{h(\text{cm})\sqrt{\epsilon_r} - 1}} \tag{11}$$

For $\epsilon_{ff}$ we have:

$$\epsilon_{ff}(f(\text{GHz})) = \epsilon_r - \frac{\epsilon_r - \epsilon_{ff}}{1 + G\left(\frac{f}{f_p}\right)^2}, \; f_p = \frac{Z_0}{8\pi h(\text{cm})} \tag{12}$$

High impedance, thin substrates $\Rightarrow$ less dispersion.

Expressions for dispersion of $Z_0$ are also available. Another problem: losses. Both dielectric and ohmic. For dielectric substrates, dielectric losses are normally $<$ conductor losses.

Further complication: quality factor Q. Calculated from $Q = \beta/(2\alpha)$ where $\beta = 2\pi/\lambda$ and $\alpha$ is the total loss.

$\Rightarrow Q = \pi/(\lambda\alpha)$, or $Q = 8.686/(\lambda\alpha)$ (in dB), or $Q = 27.3/\alpha$ (in dB/$\lambda$).

Quality factor Q also affected by radiation losses, so that the total can be expressed as:

$$\frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r} \tag{13}$$

$$Q_c = \frac{\pi}{\lambda\alpha_c},\ Q_d = \frac{\pi}{\lambda\alpha_d},\ Q_r = \frac{Z_0}{480\pi\frac{h}{\lambda_0}F} \tag{14}$$

Uses of microstrip tr. lines:

- Series tr. lines
- short and open circuited stubs
- microstrip + short/open circuited shunt stub can transform 50 Ohm resistor into any value of impedance
- quarter-wave microstrip can transform 50 Ohm resistor to any value of resistance
- quarter-wave transformer + one of the stubs = any value of impedance

## $\mu$-strip in matching networks

Using short/open-circuited stubs and tr. lines: see fig. 2.5.6.

For design procedure, see fig. 2.5.7. General idea: use shunt stub to get onto a constant SWR circle and then use tr. line to get to the required admittance.

How to get the lengths: remember what you are using, O-C or S-C stub; that determines the starting point. The movement on the O-C or S-C stub is along the $|\Gamma| = 1$ circle until you get to the required value of susceptance!. Transmission line length determined as before.

Alternatively, match any load to 50 Ohms! See fig. 2.5.6 for procedure. Now use tr. line to get onto unity constant conductance circle and then use shunt stub to get to the origin.

Another option: use $\lambda/4$ transformer in series $+ \lambda/8$ or $3\lambda/8$ S-C or O-C stubs.

Procedure: For $Y_{in} = G_{in} + jB_{in}$ use $\lambda/4$ transformer for 50 Ohm $\Rightarrow R_{in} = 1/G_{in}$. Then add appropriate B in shunt to get $B_{in}$ part. (figs. 2.5.9)

Use: $Z_{01} = \sqrt{Z_L R_{in}} = \sqrt{50 R_{in}}$. Since $3\lambda/8$ S-C stub produces $Y_{sc} = jY_{02}$ (fig. 2.5.9), use $Y_{02} = B_{in}$ (or, $Z_{02} = 1/B_{in}$). Same B produced by $\lambda/8$ of O-C stub.

For $B < 0$, use $\lambda/8$ S-C stub or $3\lambda/8$ O-C stub. (figs. 2.5.10).

Variation: use any practical value of $Z_{02}$ (i.e. $Y_{02} \neq B_{in}$), but adjust the length of the stub to get the final $B_{in}$. (fig. 2.5.11)

Example 2.5.2 !!!

Values for source and load $\Gamma$-s that give a good match are: $\Gamma_s = 0.614\angle 160°$, $\Gamma_L = 0.682\angle 97°$. (fig. 2.5.12)

- $y_s$ and $z_s$ corresponding to these $\Gamma$-s. Use ZY chart: $Y_s = 2.8 - j1.9$ and $Y_L = 0.4 - j1.05$
- Switch to Z (or Y) S-chart and locate $Y_s$ and $Y_L$
- do source side first: fig. 2.5.5 a)
- start from $\bar{Y} = 1$ ( Y = 1/50 S); go on const. conductance circle to intersection of that circle with SWR circle for $Y_s \Rightarrow$ point A.

- to get to pt. A we need change in susceptance of $+j1.55 \Rightarrow$ shunt capacitance
- for $\mu$-strip design, read off electrical length $= 0.159\lambda$ ($Y_{oc} = j\tan(\beta l) = j\tan(2\pi l/\lambda)$ )
- from A to $Y_s$ ? Go on const. $|\Gamma|$ circle (SWR circle), i.e. put a $\mu$-strip in series. Length required? Read off directly from "wavelength toward generator" chart $\Rightarrow l = 0.099\lambda$
- Same procedure for $Y_L$: 1. intersection of SWR circle for $Y_L$ with const. conductance circle = pt. B, 2. for that $l = 0.077\lambda$, 3. follow SWR circle to $Y_L \Rightarrow \beta l = 0.051\lambda$
- Difference? Susceptance needed is inductive $\bar{Y} = -j\cot\beta l$ $\Rightarrow$ short circuited stub. Last part is again series tr. line

Short-circuiting: use bypass capacitors $C_B$ (50 to 500 pF).

1. coupling capacitors $C_A$ to prevent DC from going to source and load (200 to 1000 pF). 50 Ohm lines for soldering.
2. providing DC bias to base and collector. For collector biasing simple RF choke will do ($0.077\lambda$ line is S-C). For base DC goes directly to base terminal and needs to present an O-C to AC signal from source. Accomplished with S-C, quarter-wavelength transformer with high $Z_0$ (narrowest possible line should be used)

Modification: symmetrize the stubs. Two O-C or S-C stubs in shunt must give the same susceptance $\Rightarrow$ divide the original by two, e.g. for $\Gamma_L$ use $Y = -j0.95$ instead of $Y = -j1.9$ . From S-chart: $l = 0.13\lambda$. Series $\mu$-strip stays the same.

Geometry of $\mu$-strip :

- use duroid: $\epsilon_r = 2.23, h = 0.7874mm$
- from fig. 2.5.2, for $Z_0$ = 50 Ohm $\Rightarrow W/h \approx 3$
- use eq. 2.5.11 for more accurate calculation of W/h
- for $\epsilon_{ff}$ use eq. 2.5.7 (why do we need it ?)

Final result: W= 2.42 mm, $\epsilon_{ff}$ = 1.91. To find physical length from electrical length, $\lambda = \lambda_0/\sqrt{\epsilon_{ff}}$ is needed. For f=1 GHz $\Rightarrow$ $\lambda_0 = 30$ cm $\Rightarrow \lambda = 21.71$ cm. (Quick estimate of $\lambda$ from fig. 2.53 which gives $\lambda = 21.6$ cm)

Design no. 2: use $\mu$-strip lines with different $Z_0$ .

Transform 50 Ohm to $Y_s = (2.8 - j1.9)/50 = 0.056 - j0.038$ [S]. Use $\lambda/4$ transformer to transform 50 Ohm to $1/0.056 = 17.86$ Ohm.

$Z_{\lambda/4} = Z_0^2/Z_L \Rightarrow Z_0 = \sqrt{Z \cdot Z_L} = \sqrt{17.86 \cdot 50} = 29.9$ Ohm. This provides the **real** part of the required final $Y_s$.

For imaginary part: use O-C stub $\Rightarrow Y_{oc} = jY_0 \tan\beta l$

Reminder:

$$Z_{in}(O-C) = -jZ_0 \cot\beta l, \quad Y_{in}(O-C) = jY_0 \tan\beta l \quad (15)$$

$$Z_{in}(S-C) = jZ_0 \tan\beta l, \quad Y_{in}(S-C) = -jY_0 \cot\beta l \quad (16)$$

We need negative imag. part of Y in both cases. From above eqs. we can use O-C stub with $l = 3\lambda/8$, or S-C stub with $l = \lambda/8$. Both look like a shunt inductor with $Y_{in} = -jY_0$

For O-C: $Z_0 = 1/Y_0 = 1/\text{Im}(Y_s) = 1/0.038 = 26.32$ Ohm.

Same procedure for load $Y_L = 0.008 - j0.021$ [S].

- $\lambda/4$ transformer on 50 Ohm load $\Rightarrow Z_0 = 79.1\Omega$.
- Another O-C stub with $3\lambda/8$ and $Z_0 = 1/Y_0 = 1/0.021 = 47.6\Omega$ produces required susceptance $-j0.021$

Modification: use balanced stubs. Keep the length the same ($3\lambda/8$) but double the $Z_0$ so that parallel combination produces 26.32 and 47.6 Ohm.

More examples:

- 2.5.3 - using different $Z_0$ and general load;
- 2.5.4 - using $\lambda/4 +$ transmission line (fig. 2.5.18): note that quarter-wavelength transformer works only with **real** loads, i.e. don't use it for general loads.
- 2.5.5 - reverse engineering: find $\Gamma$ for given circuit. Note on changing the substrate: $l' = l/\sqrt{\epsilon_{ff}}$, where $\epsilon_{ff}$ depends on substrate $\epsilon$ and W/h .