3.1 Smith Chart

Construction: Start with polar representation of Γ.

Idea: Γ_L, Γ_{in} on lossless lines related by simple phase change $\Rightarrow$ in polar plot going from Γ_L to Γ_{in} involves simple rotation.

$|\Gamma| \leq 1 \Rightarrow$ circle with radius=1 covers all possible Γs

Special points O-C: $\Gamma = 1\angle 0$, S-C: $\Gamma = 1\angle 180° = -1\angle 0$.

Other points: take e.g. $Z_L = 17.7 + j11.8\Omega$ on $Z_0 = 50\Omega$ line; $l = \lambda/6 \Rightarrow \Gamma_L = (Z_L - Z_0)/(Z_L + Z_0) = 0.5\angle 150°$

See fig. 3-17. Find $\Gamma_{in} = \Gamma_L \angle 150° - 2\beta l = 0.5\angle(150 - 120)°$. Phase change from $2\beta l = 2\frac{2\pi}{\lambda}\lambda/6 = 2\pi/3 = 120°$

$\Rightarrow$ Rotate Γ_L by 120° clockwise to obtain Γ_{in}!!

Γ anywhere between load and generator: rotation by $2\beta d$ in clockwise direction. d=distance from the load to the point.

Conversely, if Γ is known, CCW rotation by $2\beta d$ gives Γ_L .

Lossy line $\Rightarrow$ spiral; no need to trace it, just adjust the final magnitude by $e^{-2\alpha d}$.

Simple graphical solution for Γ transformation along tr. line.

What about impedance transformation? We'll use normalized impedances.

Direct relationship between Γ and $\bar{Z}$:

$$\bar{Z} = \frac{1+\Gamma}{1-\Gamma} \quad \text{and} \quad \bar{Y} = \frac{1-\Gamma}{1+\Gamma} \tag{1}$$

Special points: S-C, O-C. What else can we learn?

Since Z_0 is real, if Γ has angle 0° or 180° then $\bar{Z}$ is real as well $\Rightarrow$ horiz. axis in Γ plot contains real Z-s, starting with 0 on the left to ∞ on the right. For $\Gamma = 0$, Z=1, i.e. Z= Z_0 .

Origin of Γ plot is $\bar{Z} = 1$ in impedance plot. Fig. 3-18.

For each Γ we can find $\bar{Z}$. Idea: connect the points with same real part of Z! same for imaginary parts. (contour plots)

Constant real parts — a.k.a **const. resistance circles**. Fig. 3–19.

- all pass through $R = \infty$ point
- outermost circle is $\bar{R} = 0$ circle ($|\Gamma| = 1$); all impedances on it are purely reactive (0 to $\pm j\infty$).

Constant imaginary parts, a.k.a. **const. reactance lines (circles)**

- $\bar{X} = 0 =$ horizontal line through origin
- upper half $\Rightarrow$ postive reactance X $\Rightarrow$ inductive components
- lower half $\Rightarrow$ negative reactance $\Rightarrow$ capacitive components

Combine polar $|\Gamma|$ with Re(Z) and Im(Z) contours $\Rightarrow$ Smith chart (S-chart). Drop the polar circles and calculate $|\Gamma|$ via SWR or the scale at bottom

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \quad \text{or} \quad |\Gamma| = \frac{SWR-1}{SWR+1} \tag{2}$$

SWR read of from the right part of horizontal axis (between Z=1 and $Z = \infty$). Note: purely resistive loads have $SWR = \bar{R}$.

show S-chart. We need compass and ruler.

It is not even necessary to calculate $2\beta l$ (or $2\beta d$) since the scales are given in **fractions of wavelength** λ.

One 2π (or 360°) rotation corresponds to $d = \lambda/2$ since in that case $2\beta l = 360^\circ$

$\Rightarrow$ Full circle $=$ going along transmission line for a distance equal to $\lambda/2$.

Impedance and standing wave patterns repeat every $\lambda/2$ distance.

Smith chart also used as admittance chart. $\bar{Y} = 1/\bar{Z}, \bar{Y} = Y/Y_0 = YZ_0$ but $|\Gamma|$ does not change with the change from $\bar{Z}$ to $\bar{Y}$. $\Rightarrow \bar{Y}$ is 180° away from $\bar{Z}$ but on the same $|\Gamma|$ circle.

Same conclusion by looking at eqs. 1 — rotation of Γ by 180° transforms Z to Y.

Note on using S-chart as Y-chart:

- $\bar{Y} = 0 \Rightarrow$ O-C; for $\bar{Y} = \infty \Rightarrow$ S-C .
- resistance coord. $\Rightarrow$ conductance coord.
- reactance coord $\Rightarrow$ susceptance coord.
- positvie susceptance $\Rightarrow$ capacitive components
- negative susceptance $\Rightarrow$ inductive components (see chart)

- when in $\bar{Y}$ coordinates, angle of refl. coeff. (Γ) scale must be rotated by 180° (or, we have to go back to $\bar{Z}$ chart to look at Γ -s)

Two λ scales: toward generator and toward load.

Toward generator: starting from load and moving along tr. line toward generator, i.e. getting closer to the input $\Rightarrow$ clockwise (CW) rotation.

Toward load: starting point Γ_{in} and we want to determine Γ of points closer to the load $\Rightarrow$ CCW rotation.

Example 3-6

a) find $|\Gamma_L|, \phi_L, SWR$ along tr. line

1. calc. $\bar{Z}_L = Z/Z_0 = 0.3 + j0.5$; follow the lines until you find where Z_L is. Move along const. resistance circle until proper imaginary comp. is found.
2. circle through $\bar{Z}_L$ with center at origin $\Rightarrow$ circle of constant $|\Gamma|$ (SWR circle).
3. Z_L on this circle corresponds to Γ_L . $|\Gamma_L|$ is obtained from the radius, and angle by extending the line from origin through the point all the way to the outer circle ($|\Gamma| = 1$) and reading off the angle ($\phi_L = 124°$). $|\Gamma|$ read off either from the scale at the bottom or from SWR.

4. Intersection of const. $|\Gamma|$ circle with **right** part of the horizontal axis gives SWR. Z is real there and we know that $SWR = \bar{R}$. (here: SWR $= 4.2 \Rightarrow |\Gamma_L| = (SWR-1)/(SWR+1) = 0.62$)

b) Finding Γ_{in} from known Γ_L and βl means rotating along constant $|\Gamma|$ circle by $2\beta l$ (lossless line). Once Γ_{in} is know, Z_{in} is read off S-chart.

1. Plot Z_L and SWR circle (const. $|\Gamma|$)
2. draw radial line through Z_L and read off the value of λ-s toward generator ($\lambda = 0.078$; no particular meaning)
3. $\lambda_0 = 40cm$ at 750MHz, l=5.2cm $\Rightarrow l/\lambda = 5.2/40 = 0.130$. Start from 0.078, rotate CW by 0.130 to 0.208 (=0.078+0.13)

4. intersection of radial line with SWR circle give Γ_{in} and Z_{in} . $|\Gamma|$ is the same as before, only angle changed to 30°.
5. read Z_{in} off the S-chart ($\bar{Z}_{in} = 2 + j2$, $Z_{in} = 200 + j200\Omega$.

c) for l=2cm, $l/\lambda = 2/40 = 0.05$; draw a radial line, find angle. Intersection with SWR circle gives Γ(2cm) and Z(2cm). $\bar{Z} = 0.47 + j0.93$, or $Z = 47 + j93\Omega$

Suppose Y_L was given; procedure is the same: use S-chart as admittance chart to find position of Y_L — rest is the same.

Finding Y_L from Z_L is easy: it's on the opposite side of the circle where values are read off ($\bar{Y}_L = 0.9 - j1.47$; note on normalization $Y_0 = 1/Z_0 = 0.01, Y_L = 0.01\bar{Y}_L$)

To find Γ_{in} from Y_{in} rotate by 180° first!

Smith chart is a very nice visual tool. It can easily show which values of Γ and Z can be obtained and which cannot. Say, $Z_{in} = 0.7 + j0.4$ cannot be obtained in this case since that impedance is not on the SWR circle.

Also, questions like finding a maximum reactive component attainable for this load can be answered easily.

$\Rightarrow Z_{in} \approx 2.3 + j2.0$; to get that $\bar{Z}_{in}$ we need a line of (0.216 - 0.078)$= 0.138\lambda$ length.

Examples of calculations with S-chart.

ex. 3–7. Finding Z_{in} for a given circuit.

- Split it up in parts as we move on the line away from Z_L towards Z_{in} .
- pt. A: add the line delay (lossless)
- $A \to B$: add $j\omega L$ $(j30\Omega)$ to total reactance; added in series.
- $B \to C$: add $j\omega C$ $(-j200\Omega)$ in shunt
- $C \to$ in: again, phase delay along the line.

- $Z_L = 2 + j1.5$; draw SWR circle through that pt.
- rotate by 0.12λ to get to pt. B,
- add $j30/50$ to impedance at pt. B looking <u>towards</u> Z_L ; done on a **const. resistance circle** (real comp. does not change); addition is in series $\Rightarrow$ use resistances, $\Rightarrow Z_B = 1 - j1.3 + j0.6 = 1 - j0.7$
- add C in shunt; impedances in shunt don't add up $\Rightarrow$ switch to admittances; convert $\bar{Z}_B$ to $\bar{Y}_B$ by 180° rotation; $Y = 1/(-j200) = j0.005$, or normalized $\bar{Y} = j0.005 * 50 = j0.25$

- now use constant conductance circle to pt. $\bar{Y}_C = 0.67 + j0.47 + j0.25 = 0.67 + j0.72$
- phase change along the line: CW rotation on SWR circle for $0.06\lambda \Rightarrow \bar{Y}_{in} = 1.3 + j1.1$; Z_{in} obtained by $180°$ rotation $\bar{Z}_{in} = 0.45 - j0.38$
- finally, de-normalize your Z-s and Y-s.

What would happen if L and C were exchanged?

Note: you must remember what you are using: Z or Y Smith chart!

ex. 3-8: using lines with different Z_0

- normalize Z_L with $Z_{01} = 50\Omega \rightarrow \bar{Z}_L = 0.4$.
- CW rotation by $l/\lambda = 5/20 = 0.25 \rightarrow l = 0.25\lambda$.
- find $\bar{Z}$ on the input terminals of Z_{01} tr. line: $\bar{Z}_A = 2.5 \rightarrow Z_A = 125\Omega$ (note the transformation : $Z_{in} = Z_0^2/Z_L$ or $\bar{Z}_{in} = 1/\bar{Z}_L$)
- normalize with $Z_{02} = 90\Omega \rightarrow \bar{\bar{Z}}_Z = 125/90 = 1.39$
- draw new SWR circle and rotate CW by $12.8/20 = 0.64\lambda$. Note: 0.5λ is <u>one full circle</u>!
- read off $\bar{\bar{Z}}_{in} = 0.9 - j0.3$ or $Z_{in} = 81 - j27\Omega$.
- last SWR is 1.39 $\Rightarrow |\Gamma| = 0.163$; angle from Z S-chart.

What can we do with Smith chart?

1. plot $\bar{Z}$ or $\bar{Y}$
2. calculate SWR, $|\Gamma|$, $\angle\Gamma$
3. impedance transformation due to tr. line
4. $Z \rightarrow Y$ and $Y \rightarrow Z$
5. add series or shunt circuit elements at any pt. on the tr. line

Variation of $\bar{Z}, \bar{Y}$ with frequency

Purely reactive networks $\Rightarrow \partial X/\partial\omega, \partial B/\partial\omega > 0$ (figs. 3–14, 15, 16).

This translates into requirement that Z and Y curves must have a CW trend with increasing frequency. Same conclusion holds for R with X, and G with B. see fig. 3–23 for qualitative behavior.

Arrows indicate direction of increasing f.

Examine tr. line with series RLC circuit (fig. 3–24). Resonance at 1 GHz. Load is $Z_L = 0.5 + j\bar{X}$ and $\bar{X} = (\omega L - 1/(\omega C))/50$.

Calculate Z_L at resonance, above, below: see table 3–1.

CW change with freq. increase. What's the effect of tr. line?

Each Z_L has SWR circle; rotate according to table 3–1. (S-chart)

Problem: rotation is frequency dependent! $l/\lambda = 0.1, 0.133, 0.167 \Rightarrow$ proportional to f! (electrical length $\beta l = 2\pi l/\lambda = \omega l/v$)

Note: Z_{in} vs. f is more spread out on S-chart than Z_L itself!

Z_{in} covers 0.151λ while Z_L covers only 0.084λ. Diff. of 0.067λ is due to freq. change of tr. line.

Longer line (large Δf) $\Rightarrow$ larger impedance spread $\Rightarrow$ more difficult to match impedance on Δf.

Problem 3.51 for illustration.

Standing wave patterns

Use S-chart for this; idea from $\hat{V}/\hat{V}^+ = 1 + \Gamma$, $\hat{I}/\hat{I}^+ = 1 - \Gamma$

Magnitudes: $V/V^+ = |1 + \Gamma|$, $I\ I^+ = |1 - \Gamma|$

V, Γ are functions of distance; for lossless lines V^+, I^+ are not! Simple vector addition and subtraction gives the ratios (normalized rms voltage and current).

Fig. 3–25 for $Z_L = 1 + j2$. Remember $1 \equiv 1\angle 0°$. **Vector** addition (subtraction) gives V/V^+ $(I/I^+$.

Γ moves away from Γ_L on tr. line (SWR circle). Pick a few specific el. lengths and do vector summation and reconstruct SW pattern.

Max. V when $\angle\Gamma = 0$; (min. I); min. V (max I) when $\angle\Gamma = 180^\circ$.

Read position on tr. line by reading off the distance from "wavelengths toward generator" (need freq. and velocity). Fig. 3–26.

Summary of Smith-chart:

- polar plot of $\Gamma = (Z - Z_0)/(Z + Z_0)$. Z_0 is the tr. line characteristic or reference (normalizing) impedance.
- Normalized impedances: $z = Z/Z_0 \Rightarrow \Gamma = (z-1)/(z+1)$
- Normalized admittances: $y = Y/Y_0 \Rightarrow \Gamma = (y-1)/(y+1)$
- Moving along tr. line $\Rightarrow$ CW circular motion on S-chart (lossless).
- Note the positive and negative reactances/susceptances
- Constant real and imaginary impedances are on circles. Unit Γ circle is for pure reactances/susceptances.
- conversion from Z to Y done by 180° rotation.

Negative resistance results in $|\Gamma| > 1$. Where to put it on S-chart? One choice is "compressed" Smith chart showing a fraction of the negative resistance values on the horizontal axis. (fig. 2.2.4)

Alternative: plot $1/\Gamma^*$ but the resistance circles are now taken to be negative of what is read off the chart, while the reactance circles are as labeled (ex. 2.2.2).

For S-chart calculations: ex. 2.2.3 and fig. 2.2.6

ZY Smith Chart

Basic idea: Y is obtained by rotating Z in Smith chart by 180 degrees. ZY chart has them both on one piece of paper. Note: upper half is for negative susceptance, lower for positive (opposite of the Z chart). Ex. 2.3.2 and fig. 2.3.2, 3, 4.

Impedance matching networks

Why matching? For amplifiers: to deliver max. power, ensure stability, minimum noise etc. See fig. 2.4.1.

How to do it? For lower freq. use lumped elements: *ell* circuits (fig. 2.4.2). Simple procedure: ex. 2.4.1 for effects of series/shunt inductor and capacitor.

Add series reactance $\Rightarrow$ move on const.-resistance circle (fig.)

Add series susceptance $\Rightarrow$ move on const.-conductance circle.

Matching circuit design: moving from one impedance/admittance to another via const. resistance and conductance circles. Ex. 2.4.2, 2.4.3.

Not all *ell* circuits can be used for a given match: fig. 2.4.11

Quality factor Q: loaded Q defined as $Q_L = \omega_0/$ BW . ell circuits either low- or high-pass filters. Define **node** Q as: $Q_n = |X_s|/R_s$ where $R_s + jX_s$ is an equivalent series input impedance at that node. For admittance: $Q_n = |B_p|/G_p$. Fig. 2.4.13 .

How to find Q? Need ω_0 and BW. Not that simple since -3dB points are not defined. Instead, use "equivalent" bandpass filter by replacing the "input" impedance Z_B with its shunt equivalent (see fig.) Note: these are the same **only** at frequency ω_0!

$\Rightarrow$ resonant circuit loaded with 25Ω

$$Q_L = \frac{\omega_0}{\text{BW}} = \omega_0 R_T C = \frac{|B_C|}{G_T} = \frac{R_T}{|X_C|} = \frac{25}{25} = 1 \tag{3}$$

Gain of the "equivalent" circuit can be calculated and it is the same as the original one at the resonant frequency.

Real question: what is the relationship between nodal and loaded Q? In this case $Q_n = 2$ and $Q_L = 1 \Rightarrow Q_L = Q_n/2$, same for all ell. Since the BW is not really defined for the original circuit, estimate it by using $BW \approx f_0/Q_L = 500$ MHz. Alternatively, read it off the plot $\Rightarrow$ BW= 440 MHz, which is close to the estimate. Example 2.4.5.

$\Rightarrow$ to get high Q_L, high value of Q_n is needed. (note on complex loads). Problem: with ell circuits we cannot adjust Q! Solution: use three elements, i.e. T or Π networks.

Problem: Q_n and Q_L not easily related. Normally we just use the highest Q in a T or Π circuit. Useful tool: constant Q circles on Smith chart fig. 2.4.16. Center of circles at $(0, \pm 1/Q_n)$ and radius is $(1 + 1/Q_n^2)^{1/2}$.

Design example(s) 2.4.6, 2.4.7. Note that the final Q is determined from frequency plot. Also, there are many possible solutions. example 2.4.8 (why?).

Final note on amplifiers: full circuit, dc part, ac part (fig. 2.4.20). Various C-s and L-s (RFC-s) serve to decouple various part of the circuit. I/O matching done via ell circuits.