Noise

Design objective: low-noise amplification. Problem: min. noise and max. power **do not** go hand in hand! (why?)

Need design methodology for a compromise between the two. Trade-offs: noise, stability, gain.

GaAs MESFETs generally have better noise performance than Si BJTs (at high frequencies), but many new devices compete.

Problem: low noise means low current $\Rightarrow$ not good for gain!

Many sources of noise: 1) Johnson and 2) shot noise. The first one dominates in resistors and is present even when there is no current. Shot noise associated with current flow and is proportional to current.

Some basic concepts and properties:

$$\bar{v_n} = \lim_{T \to \infty} \frac{1}{T} \int_{t_1}^{t_1+T} v_n(t) dt = 0 \quad (1)$$

$$\bar{v_n^2} = \lim_{T \to \infty} \frac{1}{T} \int_{t_1}^{t_1+T} [v_n(t)]^2 dt = \text{ const.} = v_{n,rms}^2 \quad (2)$$

Often we break up a “device” into noiseless part + noise source. see fig. K1 for resistor.

In amps: no source on input but some small output voltage (power) can be measured $\Rightarrow$ amp noise power.

Resistor on input produces additional noise on output since the noise coming from resistor is amplified. Total noise power = amplified noise input power + noise output power produced by the amp.

Input need not be a resistor but we can always find an "equivalent" resistor that will produce the same noise power (or voltage). (see fig. K.2). Noise temperature T_s.

RMS value of noise voltage produced by a resistor over a frequency range $f_H - f_L = B$ is: $V_n = \sqrt{kTBR_N}$. B = bandwidth, k = Boltzmann const., T = (noise) temperature (K). fig. 4.2.1

- depends only on B, not on frequency itself; if B is the same, f does not matter ⇒ "white noise".
- $\sqrt{T}$ dependence ⇒ to reduce noise operate at lower T-s

How much power is generated? Max. available power from the resistor

$$P_N = \frac{V_N^2}{4R_N} = kTB \tag{3}$$

Example: $R_N = 2M\Omega$, T = 290 K, B = 5 kHz ⇒ $V_N = 12.6\mu V$, $P_N = 19.9 \cdot 10^{-18}$W.

Note: P_N does not depend on value of R_N but only on T and B.

Figure of merit for amps? NOISE FIGURE (F or NF) often used. Definition:

$$F = \frac{\text{total avail. noise P at amp. output}}{\text{avail. noise P at amp. output due to } R_N} = \frac{P_{No}}{P_{Ni} G_A} \quad (4)$$

recall the definition of G_A

$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{\text{P avail. from network}}{\text{P avail. from source}} \quad (5)$$

using different notation: $G_A = P_{So}/P_{Si}$ to rewrite eq. for F

$$F = \frac{P_{Si}/P_{Ni}}{P_{So}/P_{No}} = \frac{\text{avail. S/N ratio at input}}{\text{avail. S/N ratio at output}} \quad (6)$$

F measures how much noise is contributed by amp itself (not much can be done about the noise that is already on input!)

P_{N0} has two components: one from input, the other due to internal noise from amp $\Rightarrow$ these add up.

$$F = \frac{P_{N0}}{P_{Ni}G_A} = \frac{P_{N1} + P_{Ni}G_A}{P_{Ni}G_A} = 1 + \frac{P_{N1}}{P_{Ni}G_A} \tag{7}$$

remember: P_{N1} contains contrib. from amp alone and does not depend on what is on input.

Also note that second part is reduced by increasing G_A ($P_{Ni} = kTB$); problem arises if P_{N1} increases as G_A is increased.

We can also write (define $T_e = P_{N1}/(kBG_A)$):

$$P_{N0} = G_A P_{Ni} + P_{N1} = G_A k T_s B + k T_e B G_A \Rightarrow F = 1 + \frac{T_e}{T_s} \tag{8}$$

where T_e is effective input noise temperature (see fig. K.3).

What is the best value of F? If input signal is amplified, then magnitude of noise signal increases as well. However, if no new noise is added, S/N ratio on input and output are the same $\Rightarrow$ F $=$ 1 or F $=$ 0 dB. This does not happen so F increases $\Rightarrow F > 0$ dB.

Two stage amplifier noise figure

(sketch the figure) G_A-s are available poser gains at each stage; P_{N1}, P_{N2} are noise powers caused by internal ampl. noise. What's the total noise power?

$$P_{No,tot} = G_{A2}(G_{A1}P_{Ni} + P_{N1}) + P_{N2} \tag{9}$$

$$F = \frac{P_{No,tot}}{P_{Ni}G_{A1}G_{A2}} = 1 + \frac{P_{N1}}{G_{A1}P_{Ni}} \frac{P_{N2}}{P_{Ni}G_{A1}G_{A2}} = F_1 + \frac{F_2 - 1}{G_{A1}} \tag{10}$$

where $$F_1 = 1 + \frac{P_{N1}}{P_{Ni}G_{A1}}, \quad F_2 = 1 + \frac{P_{N2}}{P_{Ni}G_{A2}} \tag{11}$$

Obviously, F_1, F_2 are noise figures for each amp stage separately.

F of 2nd stage **reduced** by $G_{A1} \Rightarrow$ nice to have high gain in input stage!

Design trade-off: it may not always be best to minimize 1st stage noise if that results in large reduction of power gain since that may increase the contribution from the second part.

General conclusion: there is always a compromise between gain and noise!

Is it better to put amp1 first or amp2? The overall noise figure F for the two cases is:

$$F_{12} = F_1 + \frac{F_2 - 1}{G_{A1}}, F_{21} = F_2 + \frac{F_1 - 1}{G_{A2}} \tag{12}$$

We require $F_{12} < F_{21}$ (i.e. that 1st amp should go first) $\Rightarrow$

$$\frac{F_1 - 1}{1 - 1/G_{A1}} (= M_1) < \frac{F_2 - 1}{1 - 1/G_{A2}} (= M_2) \tag{13}$$

where M_1, M_2 are **noise measures** for two amplifiers. $\Rightarrow$ if $M_1 < M_2$ than amp1 goes first. For identical amplifiers:

$$F = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} + \ldots = 1 + \frac{F_1 - 1}{1 - 1/G_{A1}} = 1 + M_1 \tag{14}$$

Noise figure of a two-port amp can be expressed as:

$$F = F_{min} + \frac{r_n}{g_s} |Y_s - Y_0|^2 \tag{15}$$

where $r_n = R_N/Z_0$ is equiv. normalized noise resistance of the two-port, $Y_s = g_s + \; b_s$ is source admittance, $Y_0 = g_0 + \; b_0$ is source admittance that results in minimum noise figure F_{min}. After expressing Y_S, Y_0 in terms of their Γ-s

$$F = F_{min} + \frac{4r_n |\Gamma_s - \Gamma_0|^2}{(1 - |\Gamma_s|^2)|1 + \Gamma_0|^2} \tag{16}$$

Noise parameters F_{min}, r_n, Γ_0 are given by transistor manufacturers. They are determined by:

- source refl. coeff. is varied (using tuner on input terminals) until min. F is observed in noise-figure meter.
- disconnect amp and find the source refl. coeff. ($\Gamma_s = \Gamma_0$
- r_n determined when $\Gamma_s = 0$ (how is that done?)

$$r_n = (F_{\Gamma_s=0} - F_{min})\frac{|1+\Gamma_0|^2}{4|\Gamma_0|^2} \qquad (17)$$

F_{min} is function of operating current and frequency!

(time permitting derive eqs. for noise circles)

Noise figure parameter (for a given noise figure F_i)

$$N_i = \frac{|\Gamma_s - \Gamma_0|^2}{1 - |\Gamma_s|^2} = \frac{F_i - F_{min}}{4r_n}|1 + \Gamma_0|^2 \tag{18}$$

which, after some manipulation leads to

$$\left|\Gamma_s - \frac{\Gamma_0}{1 + N_i}\right|^2 = \frac{N_i^2 + N_i(1 - |\Gamma_0|^2)}{(1 + N_i)^2} \tag{19}$$

which represents circles with N_i as parameter:

$$\text{center: } C_{Fi} = \frac{\Gamma_0}{1 + N_i}, \text{ radius: } R_{F_i} = \frac{\sqrt{N_i^2 + N_i(1 - |\Gamma_0|^2)}}{1 + N_i} \tag{20}$$

For $F = F_{min} \Rightarrow N_i = 0, C_{F_{min}} = \Gamma_0, R_{F_{min}} = 0$, i.e. circle at Γ_0 with zero radius. Centers of other circles are on line from origin to Γ_0 (but $< |\Gamma_0|$).

Typical set of const. noise figure circles in fig. 4.3.2. $F_{min} = 3$ dB obtained when $\Gamma_s = \Gamma_0 = 0.58\angle 138°$.

For unilateral case design simple since const. gain circles can be drawn in the Γ_s plane and trade-off directly analyzed. As shown in Fig. 4.3.3, max. gain and min. noise figure are obtained for different Γ_s.

Example 4.3.1

Unilateral case

G_s constant gain circles drawn directly in the S-chart with noise figure circles (sketch the block diagram for amp). Remember

$$G_{TU} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \tag{21}$$

or $G_{TU} = G_S \cdot G_0 \cdot G_L$. For general analysis writeG_S, G_L as

$$G_i = \frac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2} \tag{22}$$

with radius $$R_i = \frac{\sqrt{1-g_i}(1-|S_{ii}|^2)}{1-|S_{ii}|^2(1-g_i)} \tag{23}$$

$$d_i = \sqrt{U_c^2 + V_c^2} = \frac{g_i |S_{ii}|}{1 - |S_{ii}|^2 (1 - g_i)} \tag{24}$$

For example, looking at fig. 4.3.3, $G_{s,max} = 3$ dB is obtained for $\Gamma_s = 0.7\angle 110^\circ$ (originally obtained by conjug. matching of S_{11}).

Noise figure circles drawn according to the formulas for N_i and C_{Fi}.

At Γ_s that produces F_{min}, G_s is actually less than zero dB! If we still want some gain on input, F must be increased. If max. gain is desired, then only $F > 4$ dB can be obtained.

Bilateral case

Example 4.3.1.

Calc. $\Delta = 0.41\angle 111^\circ, k = 1.012$. Unconditionally stable, but just barely. Also, not unilateral!

Using some program like UM-MAAD get the table:

dB	Center	radius
2.6	$0.45\angle 166^\circ$	0.18
2.	$0.43\angle 166^\circ$	0.25
2.8	$0.43\angle 166^\circ$	0.31
2.9	$0.43\angle 166^\circ$	0.35
3.0	$0.43\angle 166^\circ$	0.39

For (relatively large changes in Γ_s, noise figure does not change all that much. E.g. $F = 2.6$ dB circle (0.1 dB increase from $F_{min} = 2.5$ dB) results in Γ_s change of 0.2 (see table)

On the output: design for max. power (gain) $\Rightarrow \Gamma_L = \Gamma_{OUT}^*$. What to do on input?From the design requirement we have to minimize F $\Rightarrow \Gamma_s = \Gamma_0 = 0.475\angle 166°$.

With Γ_s determined, find out Γ_{OUT} and then $\Gamma_L = \Gamma_{OUT}^* = 0.844\angle 70.4°$.

Plug $\Gamma_s, \Gamma_L, \Gamma_{IN}, \Gamma_{OUT}$ into formulas for G_A, G_p

$$G_P = \frac{1}{1-|\Gamma_{IN}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = f(\Gamma_L, [S]) \tag{25}$$

$$G_A = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1}{1-|\Gamma_L|^2} = f(\Gamma_s, [S]) \tag{26}$$

$\Rightarrow G_A = 11$ dB, $G_P = 12.7$ dB. Because of conj. match on output $\Rightarrow G_T = G_A$.

Next step: design the matching networks!

Going to $\Gamma_s = 0.475\angle 166^\circ$ starting from origin (source is 50 Ohm) $\Rightarrow Z_s = 0.36 + \ \ 0.14$ and $Y_s = 2.8 - \ \ 0.8$. Use $\lambda/4$ transf. to get 0.3 from 0. Then add some reactance/susceptance to get to Z_s.

Real part: $Z_0(\lambda/4) = \sqrt{50 \cdot 50/2.8} \approx 30\Omega$.

Imaginary part: add the -j0.8 part. By using short circuited stub of $\lambda/8$ length, shunt susceptance is $- \ \ Y_0 \Rightarrow \bar{Y_0} = 0.8 \Rightarrow Y_0 = 0.8/50 \Rightarrow Z_0 = 62.5\Omega$

To find lengths: need λ. Frequency f=4 GHz. $\Rightarrow \lambda_0 = c/f = 7.5$ cm. For $\lambda/4$ line $Z_0 = 30\Omega$. From figures W/h $>$ 1. To use eq. for ϵ_{ff}, W/h must be found for given $Z_0 \Rightarrow$ (after calc. A=0. 42, B=13.2) W/h = 6.1 . Which leads to $\epsilon_{ff} = 1.97$ and $\lambda = 5.34$cm

Output matching network: transform $Z_L = 50\Omega$ to $\Gamma_L = 0.844\angle 70°$.

a) soldering area provided by a tr. line with $Z_0 = 50\Omega$.

b) The rest of the matching network is split up in order to provide some tuning capability, e.g. by changing the width of O-C stub (i.e. its Z_0) or by changing the length of the short circuited stub.

Show fig. 4.3.5.

How much could we get out in terms of gain? Calc. $G_{T,max} = 15$ dB. Since we calculated $G_A = G_T = 11$ dB, some gain was sacrificed to get the minimum noise figure. Reason: not conjugately matched simultaneously on input and output.

Show fig. 4.36 and 4.3. .

Note:

- high SWR on input
- good gain at 4 GHz
- minimum noise figure

SWR is another (possible) constraint in addition to F, gain and output power. For trade-offs, see ex. 4.3.2 and Fig. 4.3.9 (note that amp is unconditionally stable which allows calculation of $\Gamma_{Ms} and \Gamma_{ML}$). Fig. 4.3.8b has tabulated values for matching, VSWR(in), gain and F. For VSWR$< 1.8 \Rightarrow$ F increased by 0.25 dB and gain reduced by 0.5 dB. Final design in Fig. 4.3.8c.

Additional complication: if potentially unstable, stability must be checked. Another example: 4.3.2.

Three bias points (Q): minimum noise figure, linear power output, maximum gain. (explain names!)

For compromise between noise and gain, use linear power output point. Calculate matching and performance for three designs: Minimum F, max. gain, max. output power. (fig. 4.3.9) What happens as input changes from Γ_{opt} to Γ_{ms}? Fig. 4.3.10 shows that for straight line. The point chosen is around mid-way: VSWR=2.238, $G_A = 10.55$dB, F=3.14 dB. For results see fig. 4.3.11 and 4.3.12 .

Potentially unstable case: ex. 4.3.4.

First find $G_{MSG} = 27.27$ (14.36dB). Construct stability circle(s), gain circles and noise circles (fig. 4.3.13a). For $F_{min} \Rightarrow G_A = 8.9dB$, and VSWR(in)=9. Improvements? Get more gain at the expense of F, say pt. a gives 10.5 dB of gain, but F=1dB. Now VSWR(in) = 6.41 (what about VSWR(out)?). To reduce VSWR further, sacrifice some VSWR on output, but also increase gain G_A (why?). Const. VSWR(out) circles help choose Γ_L. Pick up, say pt. c (fig. 4.3.13) $\Rightarrow$ VSWR(in)=3. 1. (no info on G_T given; why?). Fig. 4.3.13c,d for final results

Ex. 4.3.5 illustrates design using G_p instead of G_A and for specified VSWR(in). Obviously, some trial and error procedure required!

Note on multistage amp: minimizing M (noise measure) is required. As long as G_A is large, minimizing each stage F (how?) will minimize overall F. If that is not the case, then some guesswork is necessary (unconditionally stable case): 1) plot Γ_{opt} and Γ_{MS} on S-chart, 2) break it up with 3-4 points, 3) Calculate M, G_A and VSWR(in) at those points, 4) select a point that gives the best compromise.

In the end: use computer for optimization. See appendix CAD 4.

Calculating overall G_A and F? Ex. 4.3.6.

For G_A sum up the gains of every stage (in dB).

For F, take into account the gain of previous stages, e.g.:

$$F_{LNA} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} \tag{27}$$

Since there is plenty of gain, only the first stage influences the final F!