Negative resistance oscillators

Terminology:

- load matching network (or resonant matching network) determines the frequency of oscillation
- terminating network is used to provide proper matching

One port negative resistance oscillators (fig. 5.2.1)

Negative resistance device represented by amplitude and frequency dependent impedance $Z_{IN}(V,\omega) = R_{IN}(V,\omega) + \ X_{IN}(V,\omega)$. $R_{IN}(V,\omega) < 0$ for some V and some ω.

Oscillator constructed by connecting the neg. resistance device to a passive load impedance $Z_L = R_L + \ X_L$

Recall stability conditions of two-ports. To make devices unconditionally stable we used resistive loading. Just because device is not unconditionally stable does not mean it is actually going to oscillate — total resistance in the loop must be negative.

By analogy: 1-port is stable if Re$[Z_{IN}(V,\omega) + Z_L(\omega) > 0$.

For network to actually oscillate the requirement is:

$$\Gamma_{IN}(V,\omega)\Gamma_L(\omega) = 1 \tag{1}$$

Explain in terms of reflected/incident waves. $|\Gamma| > 1$ means that there is an amplification of reflected signal.

Eq. 1 can be broken into two parts:

$$R_{IN}(V,\omega) + R_L = 0 \quad \text{and} \quad X_{IN}(V,\omega) + X_L(\omega) = 0 \tag{2}$$

Instability vs. oscillation: instability results in an uncontrolled increase of voltage and current and it has no steady state. Oscillation is a "stable" state (steady state), i.e. the amplitudes do not grow uncontrollably.

Define device as unstable over some freq. range (ω_1, ω_2) if $R_{IN} < 0$ on that range.

One port network is unstable for some ω_0 on (ω_1, ω_2) if the net resistance of the network is negative, i.e. $|R_{IN}(V, \omega)| > R_L$. Any excitation or noise can initiate such network to oscillate with ω_0 for which the net reactance of the network is equal to zero, i.e. $X_L(\omega_0) = X_{IN}(V, \omega_0)$

At ω_0 a growing sinusoidal current will flow through the circuit. Growth continues for as long as the resistance is negative ($|\Gamma| > 1$). This cannot go on forever and some non-linearity must bring about change in $|R_{IN}|$ so that when voltage V across it reaches some level ($V = V_0$) the net resistance reaches zero value $R_{IN}(V_0, \omega_0) + R_L = 0$.

Problem: oscillation frequency is not stable. It is determined from the condition on reactance $X_{IN}(V, \omega)$ which changes as V increases. For stable oscillation an additional condition is needed. Kurokawa came up with it for $Z_{IN}(V, \omega)$ that is essentially constant in ω around ω_0:

$$\left.\frac{\partial R_{IN}(V,\omega)}{\partial V}\right|_{V=V_0} \left.\frac{\partial X_L(\omega)}{\partial \omega}\right|_{\omega=\omega_0} - \left.\frac{\partial X_{IN}(V,\omega)}{\partial V}\right|_{V=V_0} \left.\frac{\partial R_L(\omega)}{\partial \omega}\right|_{\omega=\omega_0} > 0 \quad (3)$$

3 requirements for stable oscillations. Since $\partial R_L(\omega)/\partial\omega = 0$, eq. 3 can be simplified.

Example 5.2.1. Device is modeled as a parallel combination of G and C, $G(V) = G_M(1 - V/V_m)$. Find a load circuit (Z_L) to provide oscillation at ω_0 and calculate output power.

- $$Z_{in}(V,\omega) = R_{in} + X_{in} = -\frac{G(V)}{G^2(V) + \omega^2 C^2} + \frac{-\omega C}{G^2(V) + \omega^2 C^2} \quad (4)$$

- 1st condition:

$$R_L = \left.\frac{G(V)}{(G^2(V) + \omega^2 C^2}\right|_{\omega=\omega_0, V=V_0} \quad (5)$$

(note: this does not mean R_L is freq. dependent!)

- for X we have

$$X_L(\omega) = \left.\frac{\omega C}{(G^2(V) + \omega^2 C^2}\right|_{\omega=\omega_0, V=V_0} \tag{6}$$

- 3rd condition:

$$\left.\frac{\partial R_{IN}}{\partial V}\right|_{V=V_0} \left.\frac{\partial X_L}{\partial \omega}\right|_{\omega=\omega_0} > 0 \tag{7}$$