ME 449/549 Measurement of Pressure and Velocity
PSU MME Dept. Gerald Recktenwald
Spring 2006 gerry@me.pdx.edu
April 13, 2006 See www.me.pdx.edu/~gerry/class/ME449/notes/

1 Overview

- Review basic equations
- Review flow regimes encountered in electronic enclosures
- Pressure measurement
- Velocity measurement at a point
- Reading
  - ▷ Azar
    - ∗ Pressure measurement: Chapter 6, especially § 6.3
    - ∗ Velocity measurement: Chapter 4, especially § 4.5, 4.6, 4.8
  - ▷ Undergrad fluid mechanics book

2 Review of relevant equations

1. Pressure Units: $p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}}$

$$1\,\text{atm} = 101325\,\text{Pa} = 14.696\,\text{psi} = 760\,\text{mm Hg at } 0\,^\circ\text{C}$$

   Integrating the hydrostatic equation, $dp/dz = -\rho g$ gives $\Delta p = -\rho g \Delta z = \rho g h$ where h is the depth of the water column. This allows us to express pressure in units of depth of water column with $h = \Delta p/(\rho g)$.
   Using $\rho_{\text{water}} = 998\,\text{kg/m}^3$ at $20\,^\circ\text{C}$

$$\begin{aligned} 1\,\text{atm} &= 10.4\,\text{m H}_2\text{O} \\ 1\,\text{cm H}_2\text{O} &= 97.8\,\text{Pa} \\ 1\,\text{inch H}_2\text{O} &= 248.4\,\text{Pa} = 3.603 \times 10^{-2}\,\text{psi}0.0025\,\text{atm} \\ 1\,\text{psi} &= 27.8\,\text{inch H}_2\text{O} \end{aligned}$$

2. Bernoulli equation
   If the flow between stations 1 and 2 is
   - steady
   - incompressible

- without losses (viscous), heat or work interactions
- along a streamline

then

$$\left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_1 = \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_2$$

3. Energy equation with head loss
For steady, incompressible flow along a pipe (one inlet, one outlet)
Munson, Young and Okiishi version: (see [3]

$$\left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_2 = \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_1 - h_{L,\text{tot}} + h_p$$

White version: (see [4]

$$\left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_1 = \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_2 + h_{f,\text{tot}} - h_p$$

$h_{L,\text{tot}}$ and $h_{f,\text{tot}}$ are the total head loss. These terms include viscous and minor losses.

3 Flow Regimes Encountered in Electronic Enclosures

1. General simplifications: incompressible flow, ideal gas
Although air can be considered to be an ideal gas, we do need to correct for the effect of altitude on density. Instruments designed and constructed in Portland (for example), must be able to work in Denver, CO

2. Laminar versus turbulent
Transition from laminar to turbulent flow occurs at a critical Reynolds number, $\mathrm{Re}_{\mathrm{cr}}$, which is unique for each flow situation.

$$\mathrm{Re}_{\mathrm{D_h,cr}} \sim 2000 \quad \text{for flow in a duct}$$

3. Steady versus unsteady
Unsteady effects can be important in electronics cooling applications

   - downstream of fans
   - low Re transitional flow
   - Vortex shedding has been tried as a heat transfer enhancement, but is not used in mass-produced consumer electronics.
     - ▷ Heat transfer enhancement is accompanied by increase in pressure drop

- ▷ Optimal operation depends on tuning of geometry and flow parameters. That is hard to do with the complex and reconfigurable geometries found in mass produced consumer electronics.

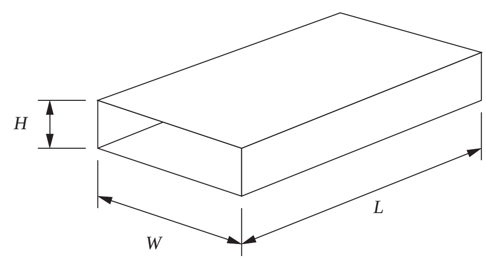
4. Fully-developed flow

   - Simple full-developed flow is steady
   - Velocity profile does not change in the flow direction, x
   - Pressure drop is linear in x direction, i.e. $dp/dx = -\text{constant}$ (True for laminar or turbulent flow.)
   - Starting from an inlet (or other disturbance) the flow asymptotically approaches a fully-developed condition. For pipes

   $$\frac{L_e}{D} \approx 0.06 Re_D \quad \text{laminar} \qquad\qquad \frac{L_e}{D} \approx 4.4 Re_D^{1/6} \quad \text{turbulent}$$

   Fully-developed flow is rarely encountered, but it still serves as an important reference case.

5. Is fully-developed flow a good model of the flow in the wind tunnels in the thermal lab?



For fully-developed flow in a duct, the flow regime is determined by the Reynolds number based on hydraulic diameter

$$\mathrm{Re}_{D_h} = \frac{V D_h}{\nu}$$

where V is the average velocity in the duct, ν is the kinematic viscosity of the fluid, and D_h is the hydraulic diameter

$$D_h = \frac{4A}{P} = \frac{2HW}{H+W}.$$

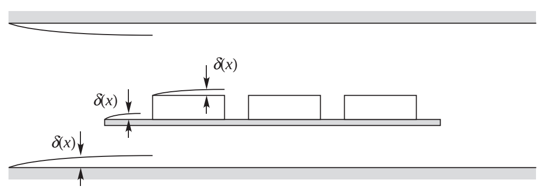
For the small wind tunnels in the thermal lab, $W = 2H$ so that $D_h = (4/3)H$. Using $\nu_{\mathrm{air}} = 1.51 \times 10^{-2}\ \mathrm{m}^2/mathrms$, and $H = 15$ cm, the following table lists values of the Re_{D_h} and the dimensionless entrance length

V (m/s)	Re_{D_h}	L_e/D_h
0.5	6711	19.1
1.0	13422	21.4
1.5	20132	22.9
2.0	26843	24.1
2.5	33554	25.0

So the flow *would be turbulent* if it were full-developed. However, the test section of the wind tunnels are only $4H = 3D_h$ long so the flow is never fully developed. Therefore, it the duct Reynolds number *is not* a good indicator of whether the flow will be turbulent in the wind tunnels.

6. Is boundary layer flow a good model of the flow in the wind tunnels?

   Boundary layers grow on all surfaces. However, there are many separation zones and recirculation zones due to the many blunt edges, and the protruding blocks (mock chips) on the (mock) circuit board.



   For boundary layers, the Reynolds number is based on x, the distance from the leading edge

$$\mathrm{Re}_x = \frac{Vx}{\nu}$$

   With a clean (sharp) leading edge, a flat plate boundary layer (parallel to the flow, no external pressure gradient) becomes turbulent at $\mathrm{Re}_x \sim 5 \times 10^5$.

   Consider boundary layer growth for the flow of air at standard temperature and pressure over a flat plate. Assume that the air is travelling at 1 m/s.

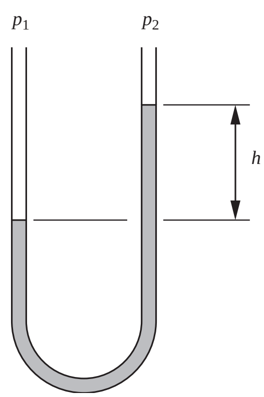
	$U_\infty = 1$ m/s	
x (cm)	Re_x	δ_{99}
10	6623	0.61
20	13245	0.87
30	19868	1.06
50	33113	1.37
60	39735	1.50
80	52980	1.74
100	66225	1.94

The data in the table suggests that the boundary layers will be laminar. In fact, due to the blunt objects and complex geometry in a typical electronics enclosure, few purely boundary layer flows will exist. The boundary layer model is still useful as indicator of the flow regime.

The boundary layer model suggests that boundary layers (when they exist) on the surfaces of objects will not be turbulent.

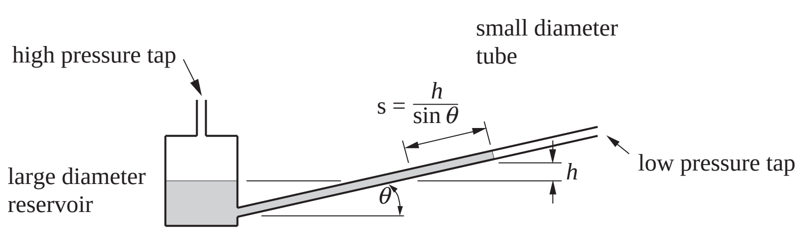
4 Instruments for Pressure Measurement

1. U-tube Manometer



$$p_1 - p_2 = \rho_m g h$$

2. Inclined Manometer



$$h = s \sin\theta \quad \Longrightarrow \quad \Delta p = \rho_m g s \sin\theta$$

The inclined manometer increases the visual resolution, allowing you to read smaller increments of pressure.

3. Bourdon gauges, e.g., Dwyer Magnehelic

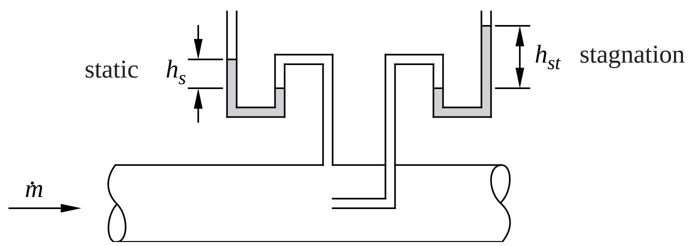
4. Electronic pressure transducers
   - Diaphragm type: useful for steady and transient
     - ▷ Capacitance transducer
     - ▷ LVDT
     - ▷ Strain gauge transducer
   - piezoelectric type: useful for high frequency transient pressure signals
5. Differential pressure transducers in the thermal lab

5 Instruments for Measuring Velocity at a Point

Three basic types of instruments for measuring velocity at a point are

- Pitot tube
- Hot wire anemometer
- Laser Doppler anemometer

5.1 Pitot Tube



If the fluid in the duct is a gas, then the pressure change due to elevation differences *in the gas* is negligible. In this case the manometer attached to the static pressure tap measures

$$p_s = \rho_m g h_s$$

where ρ_m is the density of the manometer fluid. Likewise, for a gas, the manometer attached to the Pitot stagnation pressure tube measures

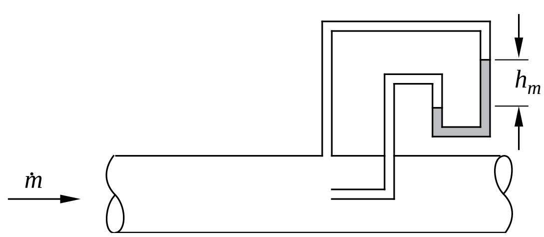
$$p_{st} = p_s + \frac{1}{2}\rho V^2 = \rho_m g h_{st}$$

Solve for V to get

$$V = \sqrt{\frac{2(p_{\rm st} - p_{\rm s})}{\rho g}} \tag{1}$$

Note that ρ is the density of the fluid flowing in the duct.

Because we are only interested in the pressure difference $p_{\rm st} - p_{\rm s}$, it is more convenient to connect the static and stagnation pressure manometers as shown below.



This arrangement is also called a Pitot probe. The manometer now measures the dynamic pressure,

$$p_d = p_{\rm st} - p_{\rm s} = \frac{1}{2}\rho V^2 \tag{2}$$

Applying the hydrostatic equation to the manometer fluid gives

$$p_{st} - p_s = \rho_m g h_m \tag{3}$$

Substituting this result into Equation (1) gives

$$V = \sqrt{\frac{2\rho_m g h_m}{\rho}} \tag{4}$$

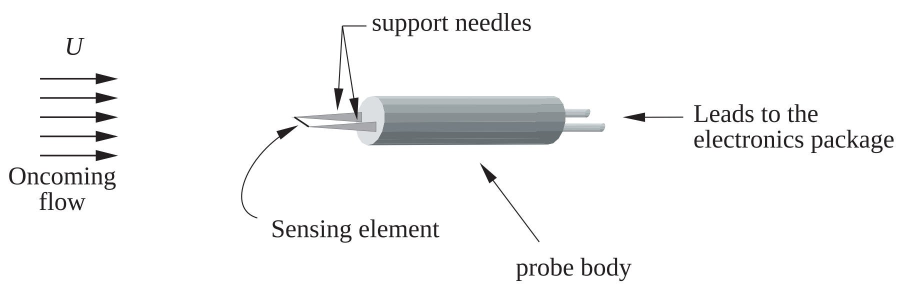
Remember that ρ_m is the density of the manometer fluid and ρ is the density of the fluid in the duct.

5.2 Hot Wire and Hot Film Anemometers

The review article by Fingerson and Freymuth [1] is a good introduction to Hot Wire Anemometry. The book by Lomas [2] is comprehensive, and it provides abundant advice to the practitioner. TSI Incorporated (St. Paul, MN) is one of the largest commercial suppliers of hot wire systems. For more information on TSI visit their web page at `www.tsi.com`.

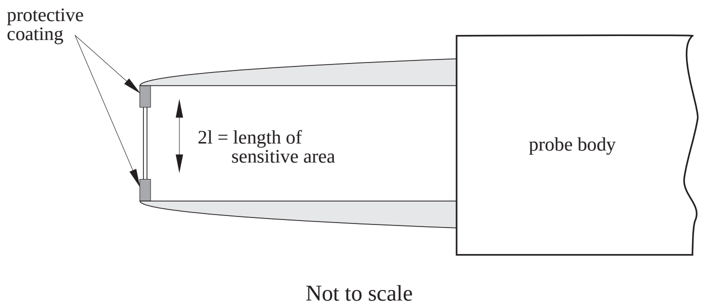
Basic Idea

The instantaneous heat transfer rate from a very fine wire is used to deduce the fluid velocity.



A typical system uses a wire made of tungsten or platinum that is approximately 1 mm long and 5 μm in diameter. The wire is attached to support needles that provide physical support and electrical connection to the output electronics. The probe can be detached from the support structure that is used to position the probe in the flow. Hot wire probes are very delicate and any planned experiment must include a budget for replacement and repair of probes.

The active sensor area is the central portion of the wire. The ends of the wire, where the wire is attached to the needle supports, are coated to provide protection and added physical support. Fingerson and Freymuth [1, § 10.2] also point out that the coating reduces measurement errors due to interference of the needle supports when the oncoming flow is not perpendicular to both the wire axis and parallel to the probe axis.



Sensor Physics

The sensor wire must have an electrical resistance that depends on its temperature. The resistance is usually assumed to be a linear function of temperature

$$R = R_r\left[1 + \alpha\left(T_m - T_r\right)\right]$$

where R_r is the resistance at T_r a reference temperature, T_m is the average wire temperature, α is the temperature coefficient of resistance for the wire. Values of α range from 8×10^{-4} °C^{-1} for an alloy of 20 percent Iridium and 80 percent Platinum to 4.5×10^{-3} °C^{-1} for Tungsten (see, e.g., [1, Table 1].

The resistance of the hot wire sensor changes in response to a change in fluid velocity. The relationship between these variables is based on the classic model of convective heat transfer from an infinitely long cylinder in a cross flow.

For *incompressible flow*

$$Q = hA\Delta T$$

$$h = f(\mathrm{Re}, \mathrm{Pr}, \theta, Gr, a_T, \frac{2\ell}{d}, \frac{k_f}{k_w})$$

where

$$\mathrm{Re} = \frac{\rho U d}{\mu} = \text{Reynolds number}$$

$\mathrm{Pr} = \dfrac{\mu c_p}{k_f}$ = Prandtl number

θ = angle between free stream velocity and normal to the wire

$Gr = \dfrac{g\beta(T_m - T_a)}{\nu^2}$ = Grashof number, an indication of the strength of natural convection

$a_T = \dfrac{T_m - T_a}{T_a}$ = overheat ratio (a.k.a. temperature loading). This parameter reflects the variation in fluid properties with temperature in the vicinity of the wire. From Figures 3.2, 3.3, and 3.4 in Lomas [2, pp. 59–60] the center of the hot wire may be as much as 250°C above the surrounding fluid temperature. In addition there is considerable temperature variation along the wire, with the ends of wire that are attached to the needle supports being at nearly the same temperature as the surrounding fluid.

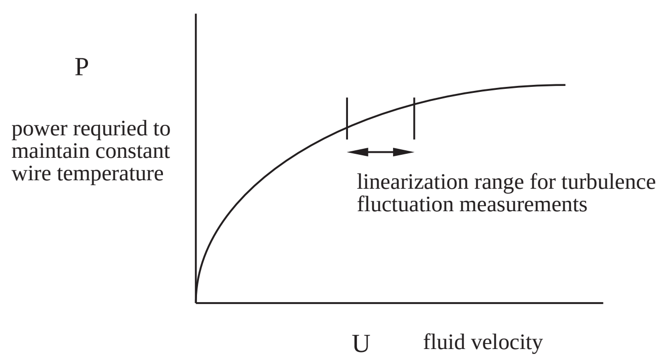
2ℓ = length of uncoated sensor wire

d = wire diameter

k_f = thermal conductivity of the fluid

k_w = thermal conductivity of the wire

In practice the preceding equation is reduced to a calibration curve for a specific sensor for a particular fluid in a given range of fluid velocities. A typical calibration curve looks like this



Each probe must be calibrated. A working formula of the form

$$E^2 = A + BV^{0.5} \tag{5}$$

is obtained, where E is the voltage drop across the sensing element, A and B are calibration constants, and V is the fluid velocity.

5.3 Electronics Package

Hot wire systems employ a Wheatstone bridge to measure the resistance of the sensor. Other circuitry is needed to control the heat transfer conditions imposed on the wire. Three modes are used [2].

1. **Constant Temperature**. A feedback circuit supplies a varying current to the probe to maintain constant wire temperature (same as constant resistance). This is the most common operation mode when the objective is to measure the instantaneous fluid velocity or the correlations of turbulence fluctuations.

2. **Constant Current**. The Wheatstone bride is designed so that resistance fluctuations in the sensing element are small compared to overall resistance between a DC power supply and ground. A constant current is therefore supplied to the wire and its instantaneous temperature (resistance) is used to indicate fluid velocity. The very first hot wire probes used constant current operation.

3. **Pulsed Wire**. Two wires are placed a known distance apart in the stream. A pulse of current temporarily heats the upstream wire and the second sensor detects the temperature rise as the fluid is convected downstream. The velocity is L/τ, where L is the distance between the wires and τ is the transit time for the pulse of heated fluid.

5.4 Ranges of Velocities Encountered

Typical velocities for forced convection air cooling are in the range 0.5 to 5 m/s (100 to 1000 ft/min). These are nominal velocities one would expect to encounter, say, in the space between two circuit boards when the air flow is driven by conventional blowers or axial flow fans. Certainly there will be dead spots in any enclosure, and in those dead spots one would find much lower velocities, including totally stagnant air. Using Equation (2) and $\rho_{\text{air}} = 1.23\,\text{kg/m}^3$ and $\rho_{\text{water}} = 998\,\text{kg/m}^3$ it is easy to compute the values of dynamic pressure and the corresponding water column height in the following table.

V (m/s)	p_d (Pa)	h (cm)	V (ft/min)	h (in)
0	0.15	0.002	98.4	0.001
1	0.62	0.006	196.9	0.002
2	2.46	0.025	393.7	0.010
3	5.54	0.057	590.6	0.022
4	9.84	0.101	787.4	0.040
5	15.4	0.157	984.3	0.062

Because the dynamic pressure values are so small, it is difficult to use a Pitot tube to measure velocities in this range shown in the table.

6 Applications of Flow Measurement

6.1 Velocity at a point

- Hard to do with precision in electronic enclosures
- Many measurements are required to get overall view of the flow
- Useful for diagnosing causes of hot spots
- Useful for calibration of CFD models, but large uncertainties are possible
  - ▷ uncertainty in position — precise location of probe is cumbersome
  - ▷ need to resolve three velocity components
  - ▷ sensor interference in tight spaces

6.2 Velocity Profiles

- Characterize uniformity of flow entering or leaving an area of interest.
- Better than knowing velocity at a point
- Requires traverse mechanism. Complete profile is time-consuming

6.3 Flow visualization

- Quick technique for diagnosing flow patterns
- Can reveal problem areas: dead spots, bypass of coolant, leakage
- Primarily for qualitative information: velocity direction not magnitude

References

[1] Leroy M. Fingerson and Peter Freymuth. Thermal anemometry. In R.J. Goldstein, editor, *Fluid Mechanics Measurements*, pages 99–154. Hemisphere Publishing Company, Washinton, 1983.

[2] Charles G. Lomas. *Fundamentals of Hot Wire Anemometry*. Cambridge University Press, Washington, 1983.

[3] Bruce R. Munson, Donald F. Young, and Theodore H. Okiishi. *Fundamentals of Fluid Mechanics*. Wiley, New York, third edition, 1998.

[4] Frank M. White. *Fuid Mechanics*. McGraw-Hill, New York, fourth edition, 1999.