Heat Transfer from a Single, Heated Block

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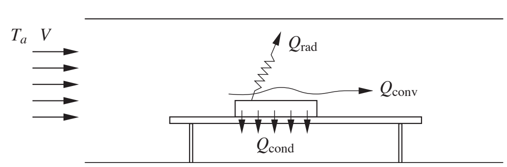
Overview

- Schematic of Basic Experiment
- Measuring the Heat Transfer Coefficient
- Heater Circuit
- Experiment Planning
- Uncertainty analysis
- Relationship to Practical Thermal Management Problems

Measurement of Heat Transfer Coefficient (1)



Calculate the heat transfer coefficient from measured quantities

$$h = \frac{Q_{\text{conv}}}{A_s(\bar{T}_s - T_a)} \tag{1}$$

where

$T_s =$ average surface temperature of the block
$T_a =$ temperature of air approaching the block
$A_s =$ surface area of block that is participating in the convection
$Q_{\text{conv}} =$ convective heat loss from the block

Measurement of Heat Transfer Coefficient (2)

Not all of the electrical power input is lost directly by convection.

An energy balance on the block gives

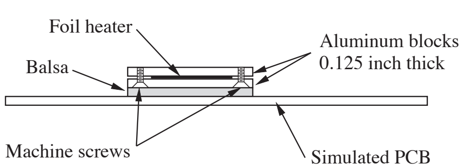
$$Q_{\text{conv}} = \dot{E}_{\text{in}} - Q_{\text{cond}} - Q_{\text{rad}} \tag{2}$$

$Q_{\text{conv}} =$ convective heat loss
$\dot{E}_{\text{in}} =$ electrical power input
$Q_{\text{cond}} =$ conduction heat transfer from block to board
$Q_{\text{rad}} =$ radiation heat transfer to surroundings

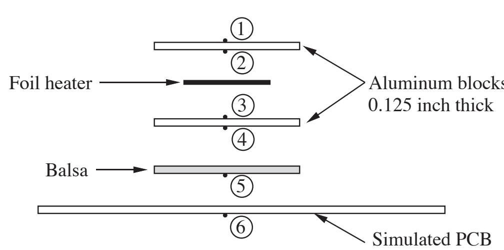
Equation (2) is a centering correction.

Device Mock-up

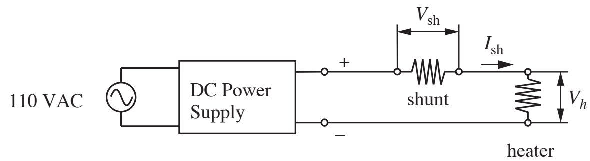
Heater assembly:



Exploded view with potential temperature measurement locations:



Measuring Power Input (1)



The power dissipated by the electric resistance heater is

$$\dot{E}_{\text{in}} = V_h I_h \tag{3}$$

where V_h is the voltage across the heater, and I_h is the current through the heater. The shunt resistor is used to measure I_h. From a measurement of V_{sh} we compute

$$I_h = I_{\text{sh}} = \frac{V_{\text{sh}}}{R_{\text{sh}}} \qquad \Longrightarrow \qquad \dot{E}_{\text{in}} = \frac{V_h V_{\text{sh}}}{R_{\text{sh}}} \tag{4}$$

Measuring Power Input (2)

Measuring power with a shunt resistor is much more accurate than measuring just the voltage across the heater and computing $\dot{E}_{\text{in}} = V_h^2/R_h$ because R_h varies with temperature.

The shunt resistor has a very low and stable resistance ($R_{\text{sh}} \sim 0.005\,\Omega$). Since R_{sh} is very small, the shunt resistor dissipates very little power. Thus its temperature will be stable and its resistance will not change during the experiment.

Measuring Power Input (3)

Sample Power Calculation:

Assume that $R_{\text{sh}} = 0.005\,\Omega$ and that $V_{\text{sh}} = 2.8 \times 10^{-3}$ V and $V_h = 8.5$ V were measured. Then

$$\dot{E}_{\text{in}} = \frac{(8.5)(2.8 \times 10^{-3})}{0.005} = 4.76\,\text{W}$$

and

$$E_{\text{shunt}} = \frac{V_{\text{shunt}}^2}{R_{\text{sh}}}$$

$$= \frac{(2.8 \times 10^{-3})^2}{0.005} = 1.57 \times 10^{-3}\,\text{W}$$

Experiment Planning

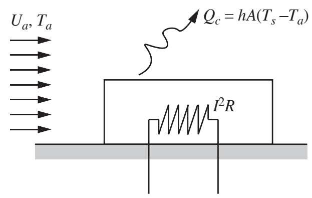
1. Decide which type of sensors, and how many of each type are needed to measure all quantities used to compute h from

$$h = \frac{Q_{\text{conv}}}{A(\bar{T}_s - T_a)}$$

2. Develop a wiring diagram for your experiment.
3. Fabricate thermocouples and attach them to your apparatus.
4. Mount your apparatus to a bottom hatch for the wind tunnel.
5. Route the sensor leads to a terminal strip. Label all wires and document the labeling scheme in your notebook and your report.

Model of Start-up Transient (1)

How long will you have to wait for steady state?



Use a simplified model to estimate the time it takes the block to reach its steady state temperature.

Assumptions:

- Block has uniform temperature
- All heat is lost by convection

Model of Start-up Transient (2)

A transient energy balance on the block gives

$$mc\frac{dT}{dt} = \dot{E}_{\text{in}} - hA(T - T_a) \tag{5}$$

where $\dot{E}_{\text{in}}$ is the electrical power input, h is the heat transfer coefficient. Use a shifted temperature θ, and the response time τ

$$\theta = T - T_a \qquad \tau = \frac{mc}{hA} \tag{6}$$

to transform Equation (5) to dimensionless form

$$\frac{d\theta}{dt} = \frac{\dot{E}_{\text{in}}}{mc} - \frac{\theta}{\tau} \tag{7}$$

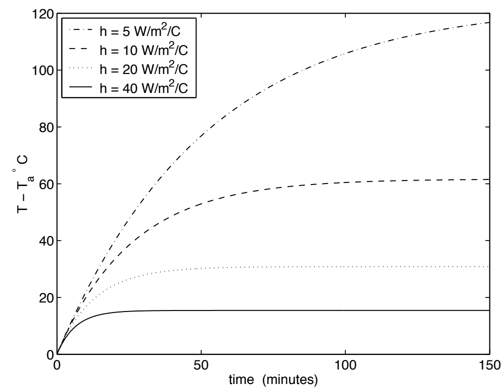
Model of Start-up Transient (3)

With the initial condition

$$\theta(t=0) = \theta_i = T_i - T_a$$

the solution to Equation (7) is

$$\theta = \frac{\dot{E}_{\rm in}}{hA} + \left(\theta_i - \frac{\dot{E}_{\rm in}}{hA}\right) e^{-t/\tau}$$



Model of Start-up Transient (4)

Define $T_{\rm ss}$ as the steady state temperature

$$T_{\rm ss} = T_a + \frac{\dot{E}_{\rm in}}{hA}$$

and t_p as the time for block to reach p percent of its final temperature difference

$$\frac{T - T_a}{T_{\rm ss} - T_a} = \frac{p}{100}$$

With our model we compute

h ($\mathrm{W/m^2/°C}$)	τ (min)	$T_{\rm ss}$ (°C)	t_{50} (min)	t_{90} (min)	t_{99} (min)
5	51	143	36	118	236
10	26	82	18	59	118
20	13	51	9	30	59
40	6	35	4	15	30

Uncertainty Analysis (1)

Suppose that your data reduction equation had the form

$$h = f(V_h, V_{\mathrm{sh}}, R_{\mathrm{sh}}, A_s, T_a, T_w, T_{\mathrm{bt}}, T_{\mathrm{bb}}, k_b, \varepsilon) \quad (8)$$

$V_h =$	voltage across the heater
$V_{\mathrm{sh}} =$	voltage across the shunt resistor
$R_{\mathrm{sh}} =$	resistance of shunt resistor
$A_s =$	surface area of the block
$T_a =$	temperature of oncoming air
$T_w =$	temperature of duct walls
$T_{\mathrm{bt}} =$	temperature on the top surface of the board
$T_{\mathrm{bb}} =$	temperature on the bottom surface of the board
$k_b =$	thermal conductivity of the board
$\varepsilon =$	emissivity of the heated block

Data analysis involves creating a routine to compute h, and perturbation of all inputs to estimate u_h.

Uncertainty Analysis (2)

Given the data reduction formula in Equation (8)

$$u_{h_{V_h}} = f(V_h + u_{V_h}, V_{\mathrm{sh}}, R_{\mathrm{sh}}, A_s, T_a, T_w, T_{\mathrm{bt}}, T_{\mathrm{bb}}, k_b, \varepsilon) - h$$

$$u_{h_{V_{\mathrm{sh}}}} = f(V_h, V_{\mathrm{sh}} + u_{V_{\mathrm{sh}}}, R_{\mathrm{sh}}, A_s, T_a, T_w, T_{\mathrm{bt}}, T_{\mathrm{bb}}, k_b, \varepsilon) - h$$

$$\vdots$$

$$u_h = \sqrt{(u_{h_{V_h}})^2 + (u_{h_{V_{\mathrm{sh}}}})^2 + \ldots}$$

where

u_{V_h}	the *total* uncertainty in the heater voltage
$u_{V_{\mathrm{sh}}}$	the *total* uncertainty in the voltage across the shunt resistor
$\vdots$	

u_{V_h}, $u_{V_{\mathrm{sh}}}$, etc. depend on random error, instrument error, and calibration error.

Shakedown

An end-to-end calibration is also called *first order replication*

- Used to verify equipment
- Make measurements of known quantities
  - ▷ Run experiment with power off: All temperature readings should be the same, i.e., room temperature
  - ▷ Perform benchmark experiments: Can you repeat experiments and obtain the same results published by others?
- One outcome is an estimate of random component of fixed error
  - ▷ Run the no-power experiment
  - ▷ Wait for system to come into steady state
  - ▷ Over time period of typical data collection, take 30 (or more) samples from each sensor
  - ▷ Compute average and standard deviation of sensor values
  - ▷ $u_{x_{\text{random}}} = 2\sigma$ for 20:1 odds (95 % confidence)

Relationship to Practical Problems

- Heaters as Prototype Devices
- Develop experience with multimode heat transfer
- Gain familiarity with complexity of measurements
- Gain experience building a thermal mock-up

References

[1] R. J. Moffat. Uncertainty analysis. In K. Azar, editor, *Thermal Measurements in Electronic Cooling*, pages 45–80. CRC Press, Boca Raton, FL, 1997.