

# The physical basis of digital computing

Henk van Houten

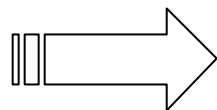
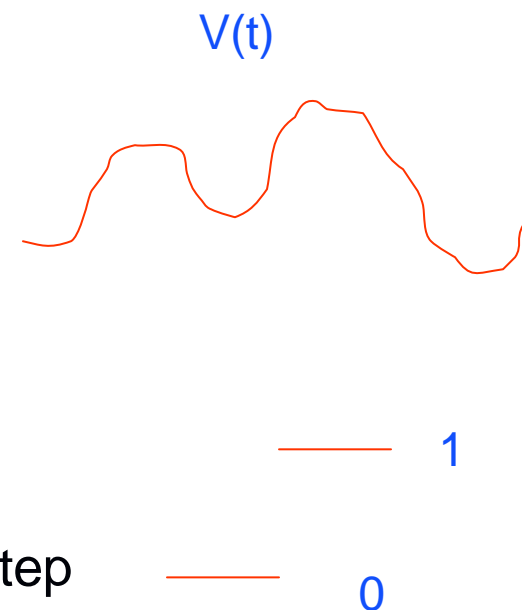
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- Digital switching and the thermal limit
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# Digital signal representation

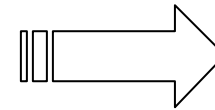
- An analogue signal can represent many bits
  - but signal distortion is inevitable in complex systems
- A digital signal represents only a single bit (0,1)
  - because no system is perfect (noise, distortion)
  - simple standardization of signal levels
  - logic level restoration possible at each computational step



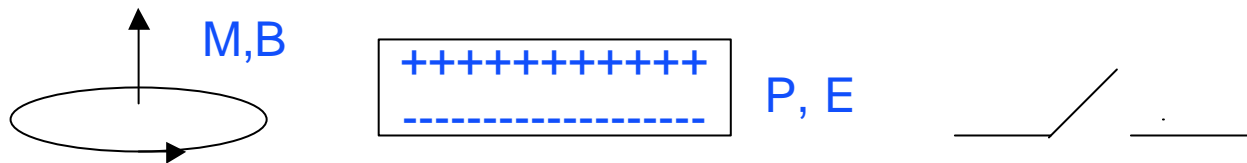
Indefinite extension possible  
without error propagation

# Digital computing has a physical basis

- computing is a physical process
  - logic devices have a finite physical extent
  - require a minimum time to perform their function
  - dissipate a switching energy
- The physical basis sets the scale for size, speed, and power requirements of a computing system
- Many physical implementations are possible



thermodynamics  
electromagnetism  
quantum mechanics  
& information theory



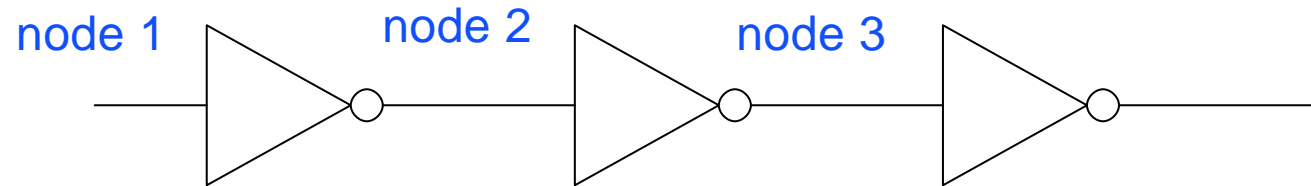
Spin, magnetization, field direction, polarization, mechanical switch  
MOSFET channel conductance,...

# Restoring logic devices must have gain

A digital signal is stored as a *signal energy*

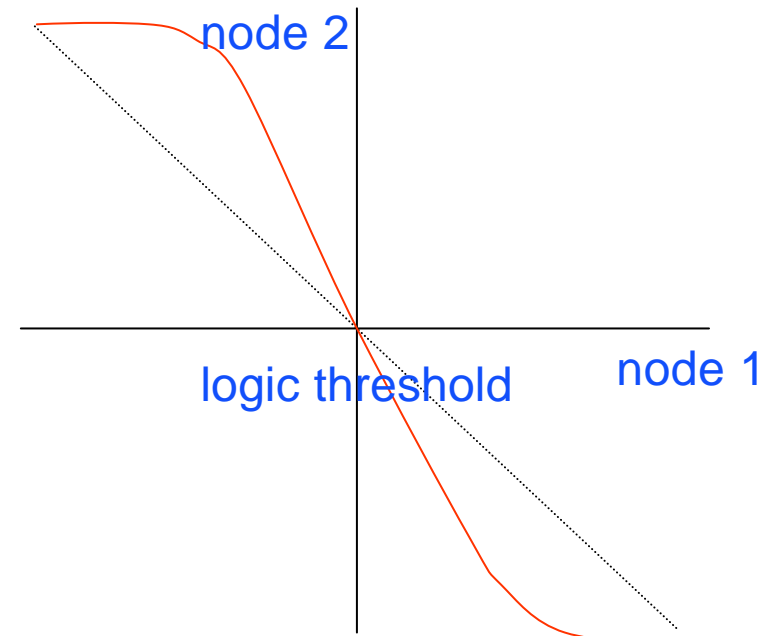
A logic circuit must be able to drive similar circuits

Inverter chain



Logic transfer characteristic in valid range for “0” or “1” the slope must be *less* than 1

so: around logic threshold the slope must be *larger* than 1

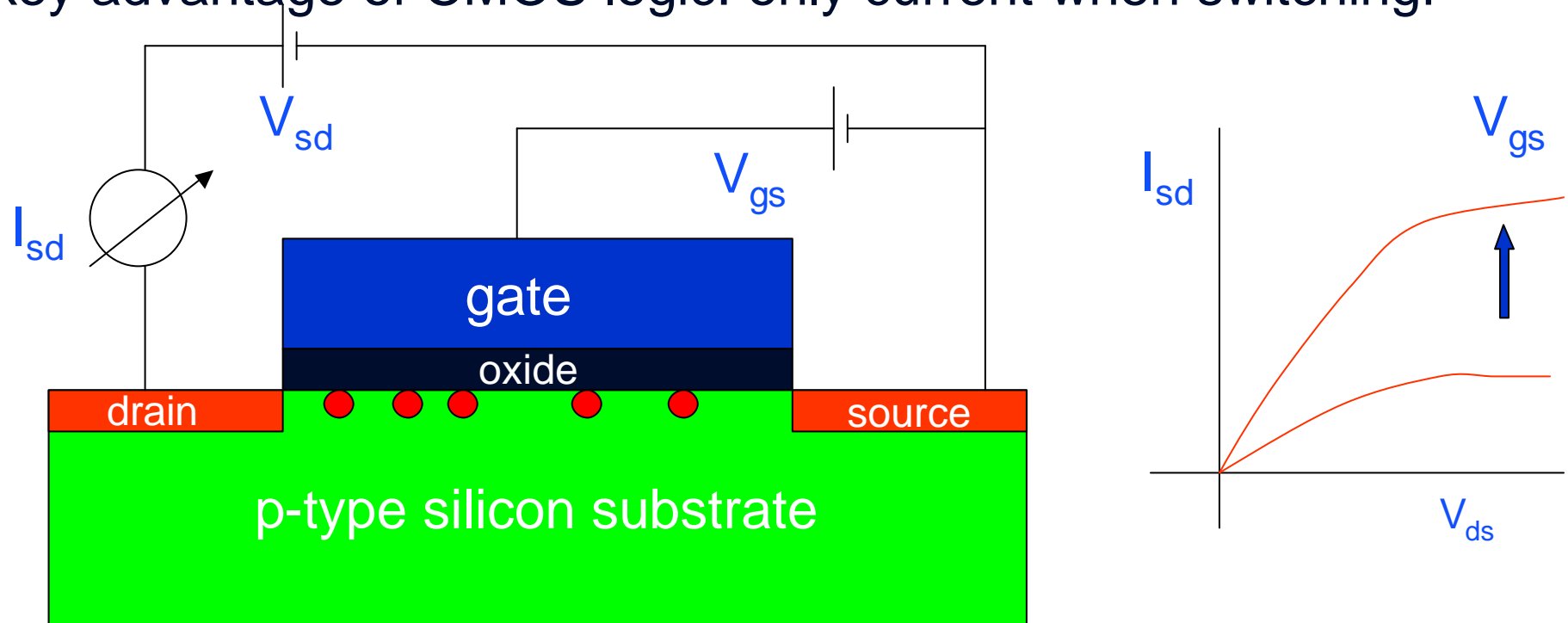


Circuit must have energy gain  
Switch in finite time: power gain

Computer must have a power supply  
separate from the signal path

# The MOSFET

- A MOSFET is basically a switchable resistor with gain
- the charge in the channel is determined by the gate voltage
- key advantage of CMOS logic: only current when switching!



Induced charge density in the n-channel  $en_{induced} = C(V_{gs} - V_t)$

# Switching time

Current through a MOSFET  
(small source-drain voltage)

$$I = e n_{\text{induced}} v_{\text{drift}}$$

Transit time

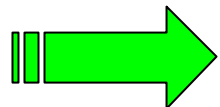
$$\tau = L/v_{\text{drift}} = L^2 / \mu V_{ds}$$

Saturation occurs when  $V_{ds} \approx V_g - V_t$

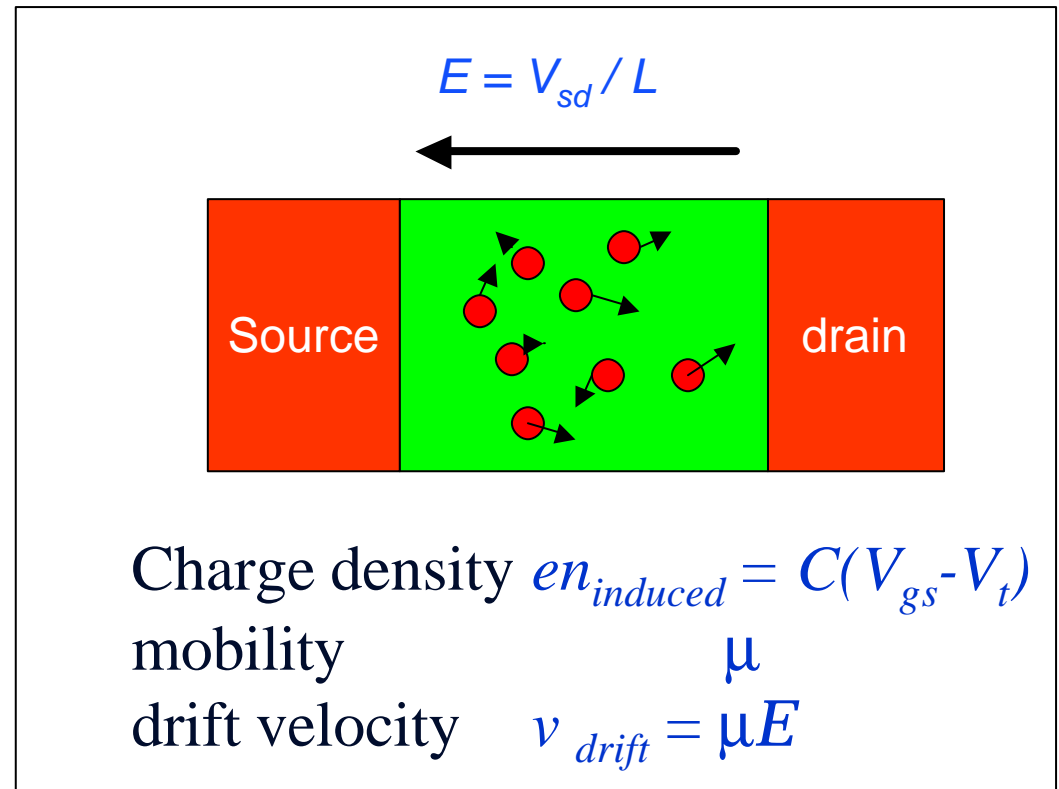
Beyond this point, the transit time no longer decreases.

Switching time is essentially the charging time of the next gate

$$RC \gg \tau$$

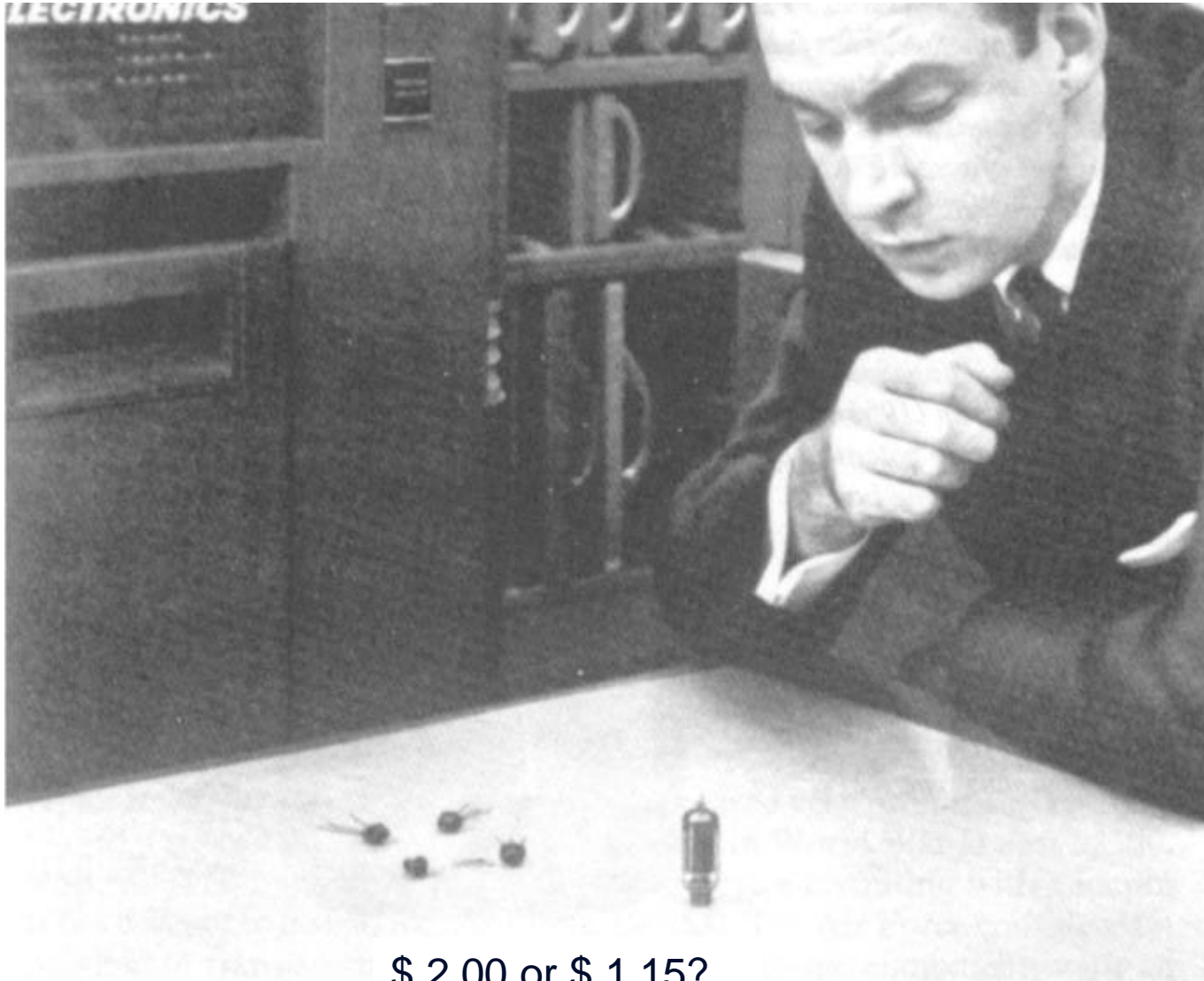


The transit time is the basic measure of switching delay



# A worried manager: miniaturization

*Advertisement of General Electric in Scientific American 1961*



\$ 2.00 or \$ 1.15?



# Moore's law: the number of transistors on a chip doubles every 12 (18) months

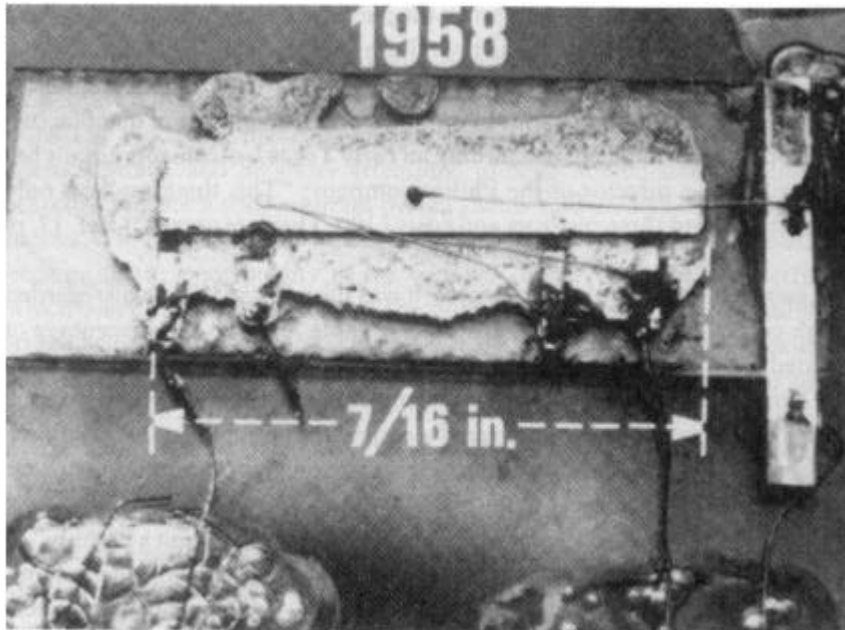
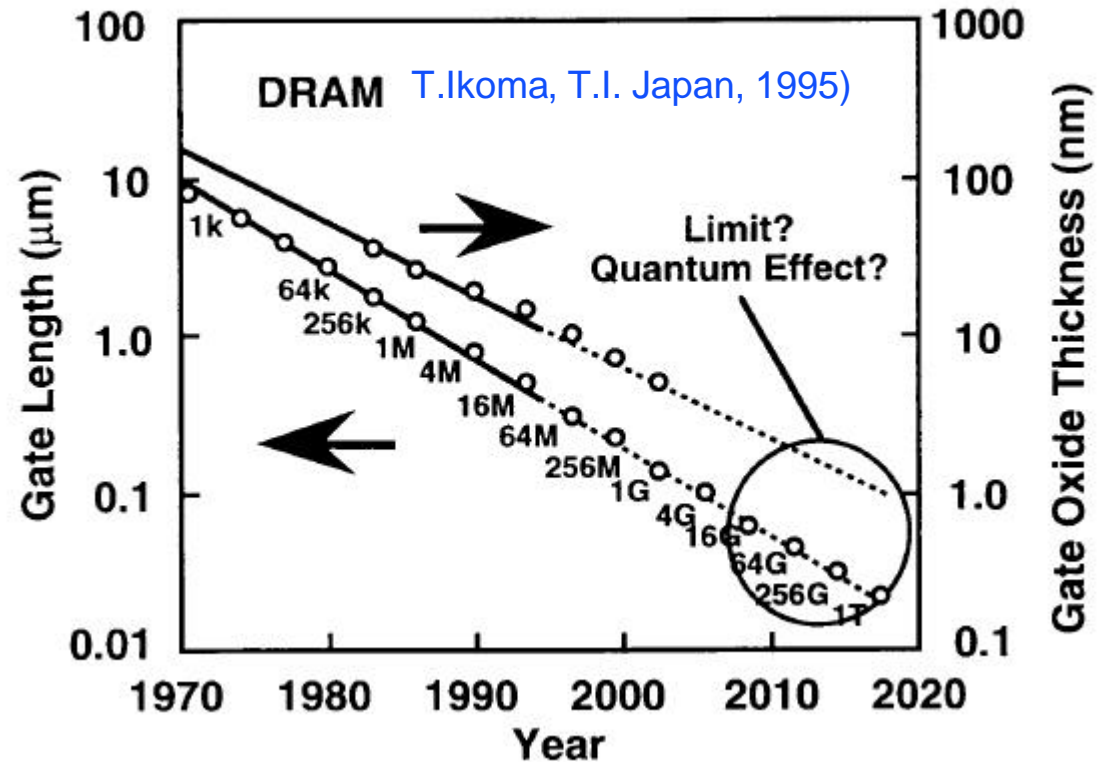


FIG. 64. The first integrated circuit, built by Texas Instruments on the basis of Kilby's sketch. ( $\frac{7}{16}$  in. equals about 11 mm).



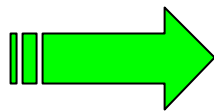
in 10 years  $\alpha \approx 5$  (Moore's law)

$$L \Rightarrow L/\alpha$$

$$d_{\text{oxide}} \Rightarrow d_{\text{oxide}}/\alpha$$

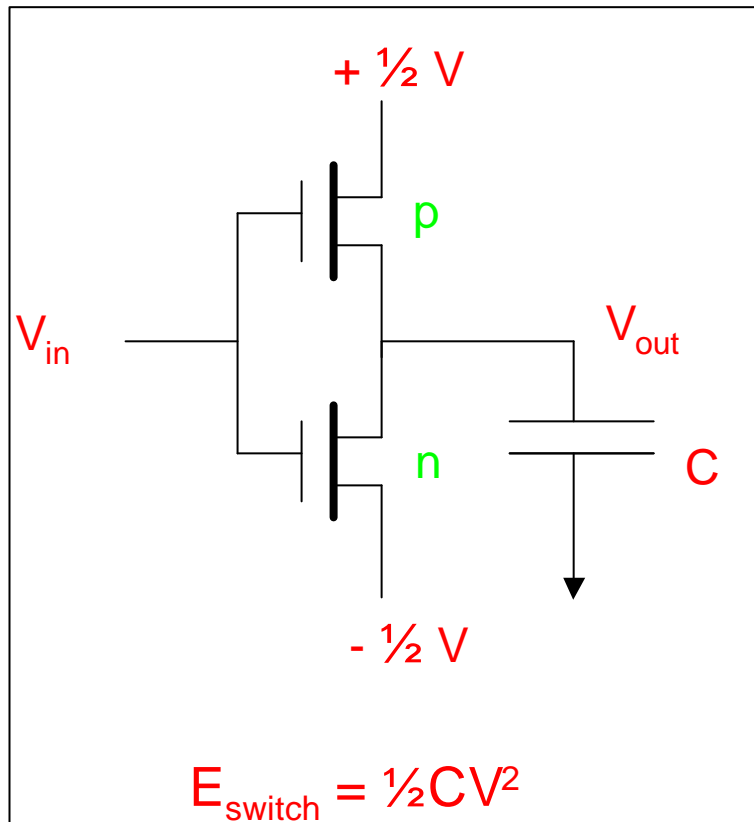
# Scaling of a MOSFET

Scale all geometrical dimensions down by  $\alpha$   
and keep the electric fields constant

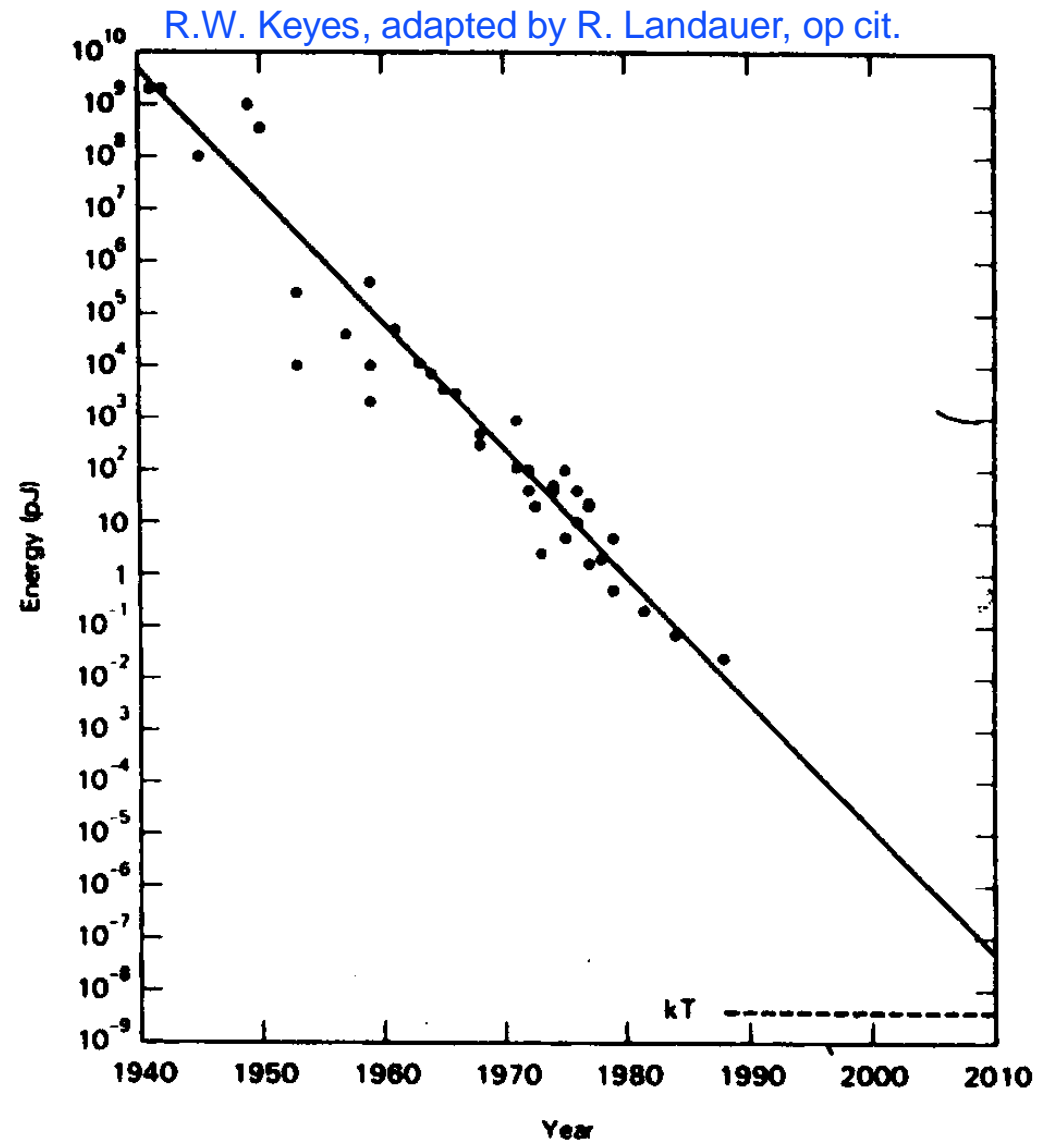


gate capacitance	$C = \epsilon W L / d_{\text{oxide}}$	$\propto 1/\alpha$
switching energy	$E = 1/2 C V^2$	$\propto 1/\alpha^3$
switching time	$\tau \propto L^2/V$	$\propto 1/\alpha$
switching power	$P = E/\tau$	$\propto 1/\alpha^2$
Power per unit area remains constant		

# Scaling of switching energy



Scaling :  $E = \frac{1}{2} C V^2 \propto 1/\alpha^3$   
in 10 years  $\alpha \approx 5$  (Moore's law)



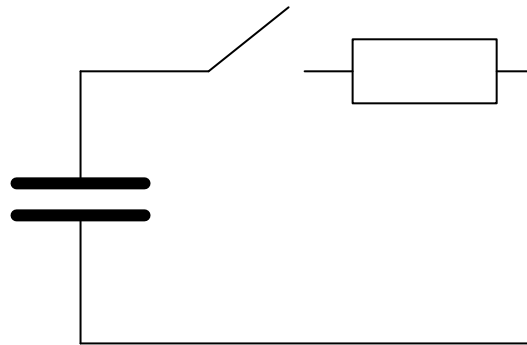
# Quasi-adiabatic computing

Can the switching energy be reduced below  $\frac{1}{2} C V^2$  ?

Yes, using “adiabatic” logic.

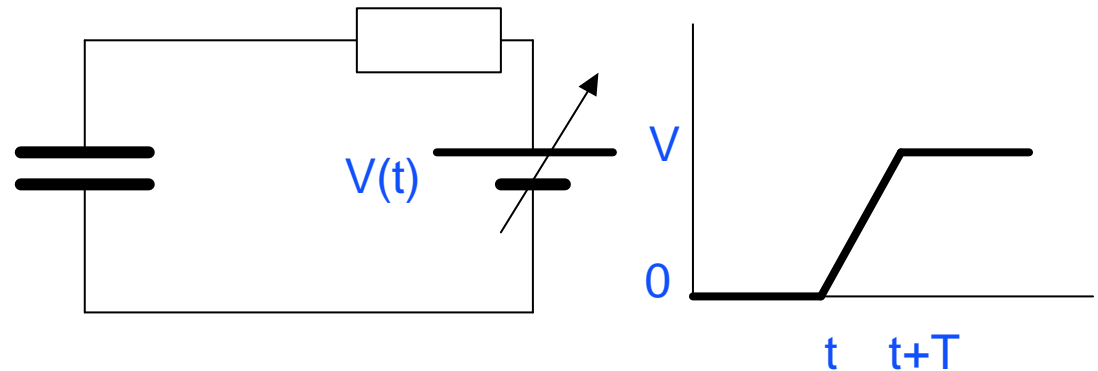
Illustration of quasi-adiabatic discharging

Discharging over a resistance



Energy dissipated is  $\frac{1}{2} C V^2$

Discharging at constant current (ramped power supply)



Energy dissipated is  $\frac{1}{2} C V^2(2RC/T)$

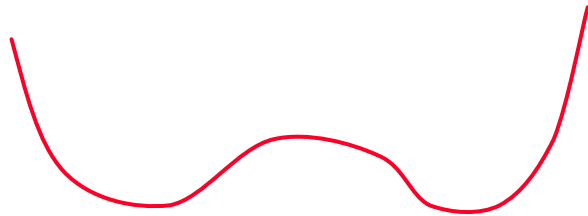
Power dissipation may be reduced at the cost of switching speed

The energy saved is stored in the power supply.

but: reduction of supply voltage has similar effect (and seems more practical)

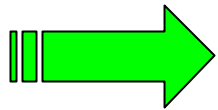
# How small can the dissipation be?

Why does the energy of a logic gate (and therefore of a digital computer) have to depend on its logical state?



Because otherwise there is no force to drive the transition, or to maintain the stable state!

Classical physics: thermal equilibrium noise  $4kTRB$  induces transitions; switching energy should exceed  $kT$



To drive a (number of) transitions in a deterministic and irreversible fashion, energy *must* be dissipated in the environment to which the system is coupled

# Side step: Communication

- Shannon: a minimal energy is associated with the transmission of a bit over a perfect channel in the presence of (white Gaussian additive) noise

$$C = B \ln [(P+N)/N]$$

{	C	channel capacity
	P	received power
	B	bandwidth
	N = kT B	noise power

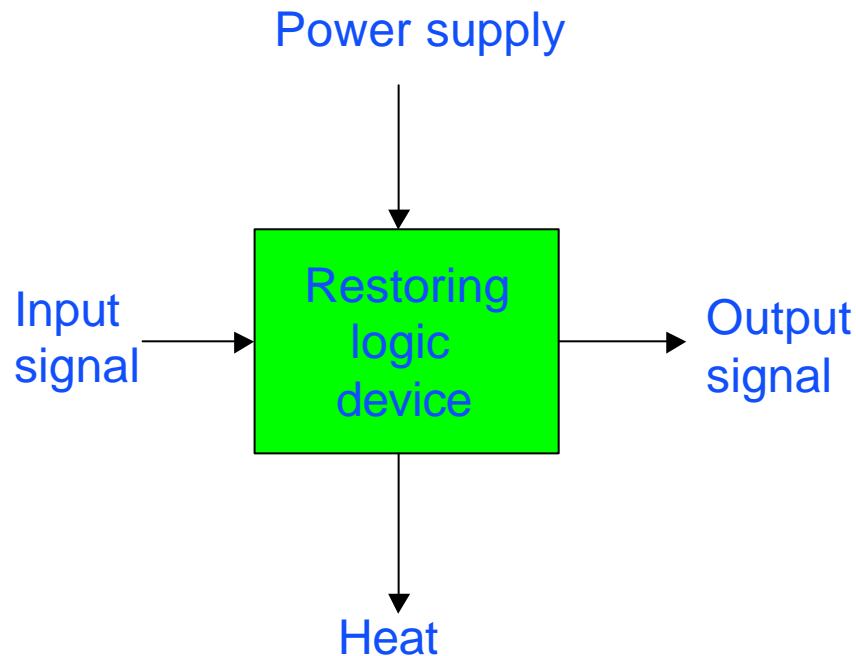
- P/C has a minimum value of  $kT \ln 2$

Landauer:

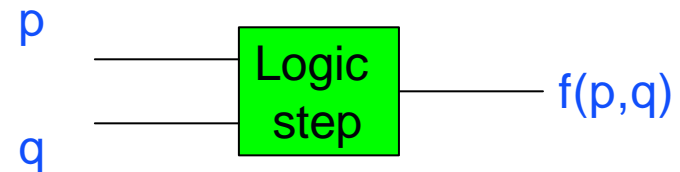
note that it is not clear that this energy has to be dissipated

note that this is an analysis of a specific case

# The second law of thermodynamics and the arrow of time



Digital computing typically involves discarding of information (Landauer)



$p, q$  can not be recovered uniquely from the non-linear function  $f(p,q)$

In a statistical sense, an *irreversible* computational process can only be driven forward deterministically at the cost of a *minimal* energy dissipation of  $k_B T \ln 2$  per erased bit (entropy drop of  $k_B \ln 2$  per erased bit)

# Reversible computers

- Information does not have to be discarded: use a series of 1: 1 mappings
- This is an extension of the “quasi-adiabatic” principle discussed before to the fully adiabatic case
- classical *reversible* circuit architectures have been proposed (Bennett, Fredkin, ...)
- the price to pay: speed, HW complexity, the system has to be flawless, ...
- This has been the basis for the field of quantum computing, see next lecture by Gianni Blatter



# Is a switching energy of $kT$ enough?

probability of error: Boltzmann factor

$$P \approx \exp(-E_{\text{switching}}/kT)$$

$$E_{\text{switching}} > kT \ln(\text{mean time between error} / \tau)$$

practical limit probably closer to 100  $kT$

(other error sources: cosmic radiation, synchronization error, etc)

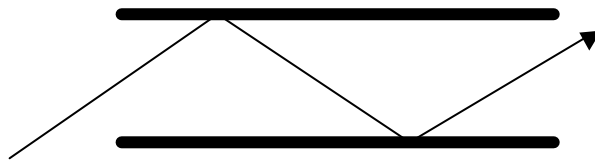
# Contents

- Digital switching and the thermal limit
- **Digital switching and the quantum limit**
- A hierarchy of limits (Meindl's classification)

# Quantum ballistic transport

*Ballistic transport* occurs on length scales short compared to the mean free path

*Specular reflection* off the boundaries of a conducting channel



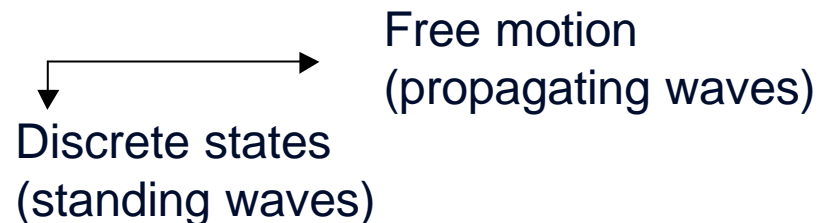
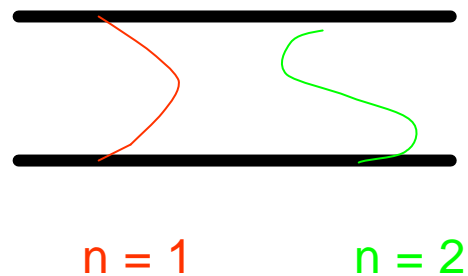
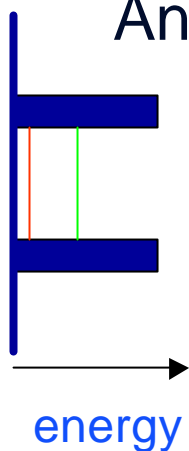
Infinite conductance?

The De Broglie *wavelength* of a conduction electron

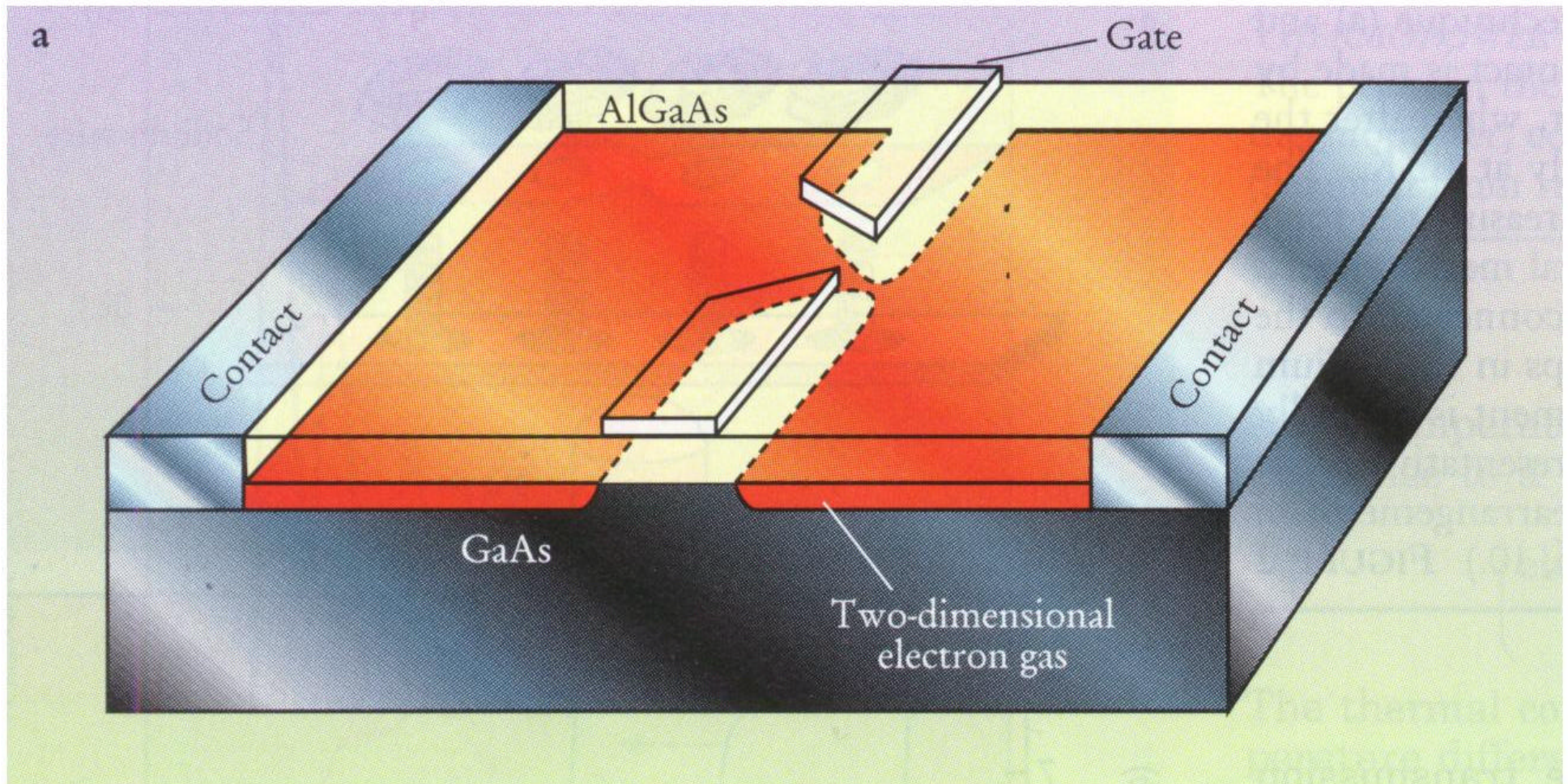
$$\lambda = h / m v_F$$

Typically 50 nm in a MOSFET or GaAs FET

An electron waveguide has 1-dimensional subbands



# The quantum point contact: a solid state electron waveguide



*Henk van Houten and Carlo Beenakker, Physics Today, July 1996, p. 22-27*

*Henk van Houten, LATSIS symposium, June 2000*

# Conductance quantization

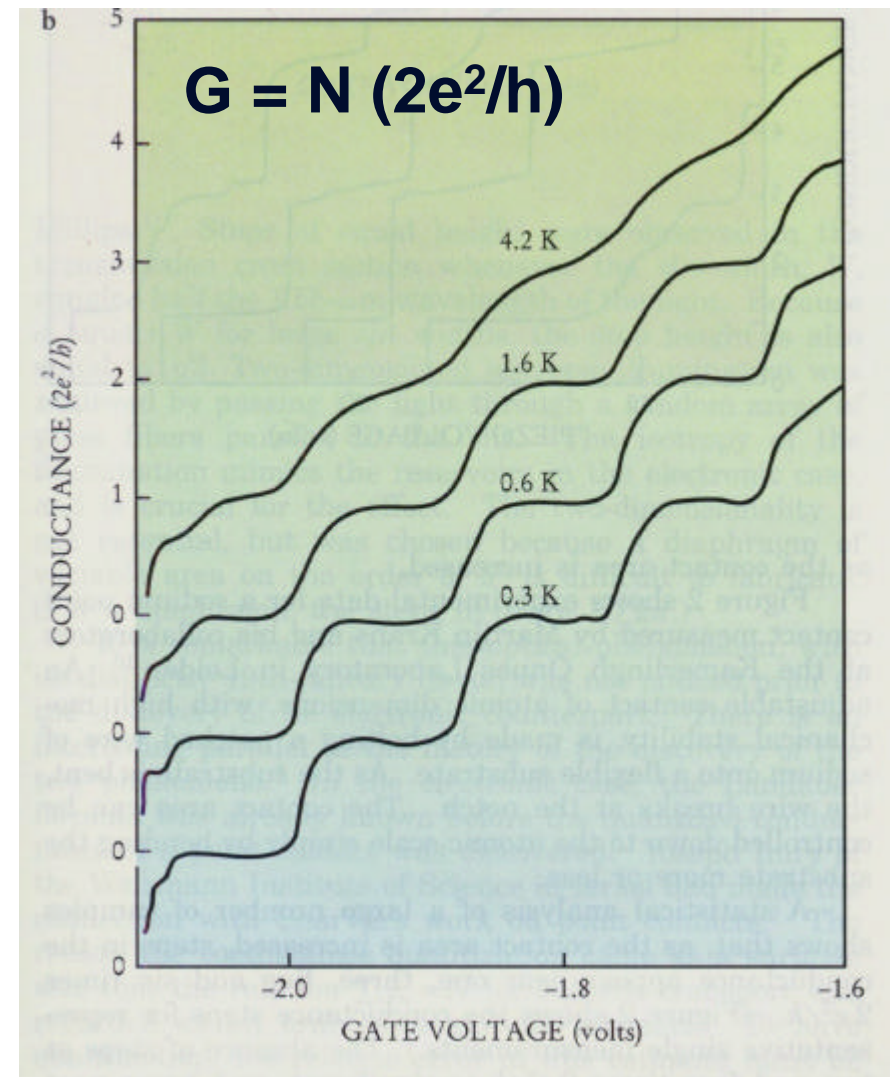
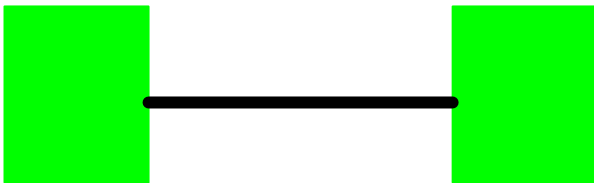
*B.J. van Wees, H. van Houten, et al. Phys.Rev.Lett. 60, 848 (1988)*

electron waveguide:

each occupied 1-d subband is a propagating mode and contributes  $(2e^2/h)$  to the conductance

quantum ballistic transport:

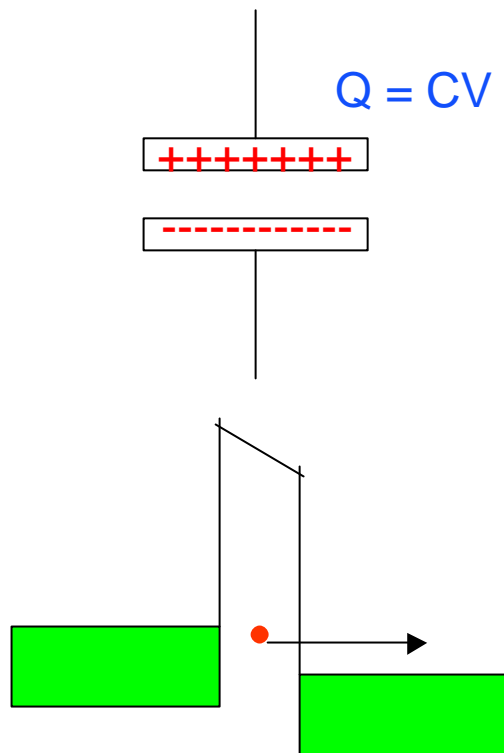
visible if the 1d subbands are separated by more than  $kT$



$e^2/h$  is a fundamental unit of conductance (cf. quantum Hall effect), it is the conductance of a single mode propagating from one **reservoir** to another

# Single electron tunneling

Millikan 1911: charge is quantized, the elementary charge is  $e$

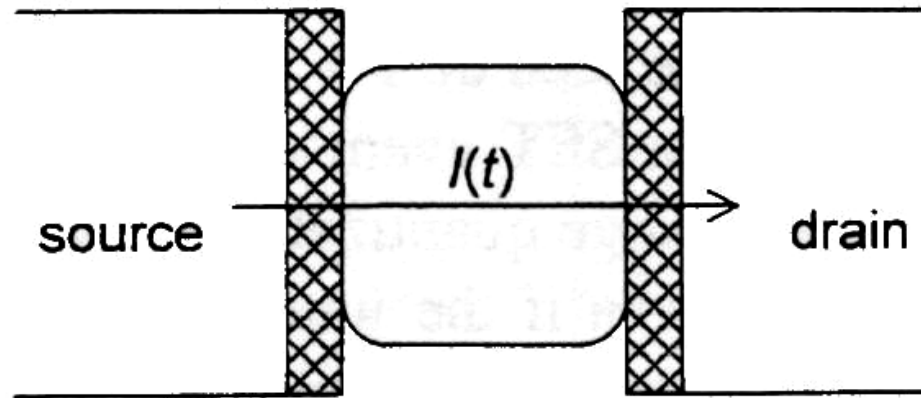


the charge induced on a capacitor can have any (fractional) value...

... but tunneling of electrons is a discrete process (manifested as shot noise with power  $2e I$ )

# Single electron tunneling

A metallic island is coupled by tunnel barriers to metallic leads.  
The capacitance of the island with respect to the environment is  $C$

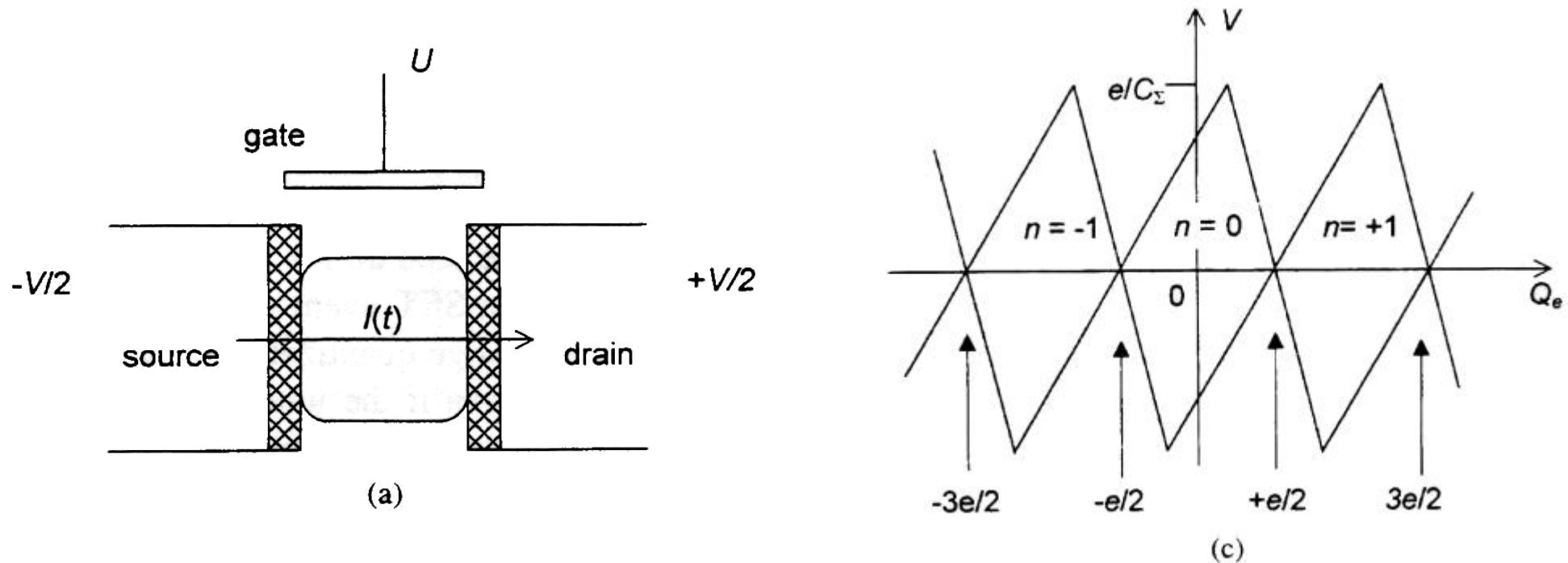


Number of electrons on an island is an integer if

- the elementary Coulomb charging energy  $e^2/C \gg kT$
- coupling to source and drain through tunnel barriers with resistance  $R \gg h/e^2$

**Coulomb blockade: no tunneling for small voltage!**

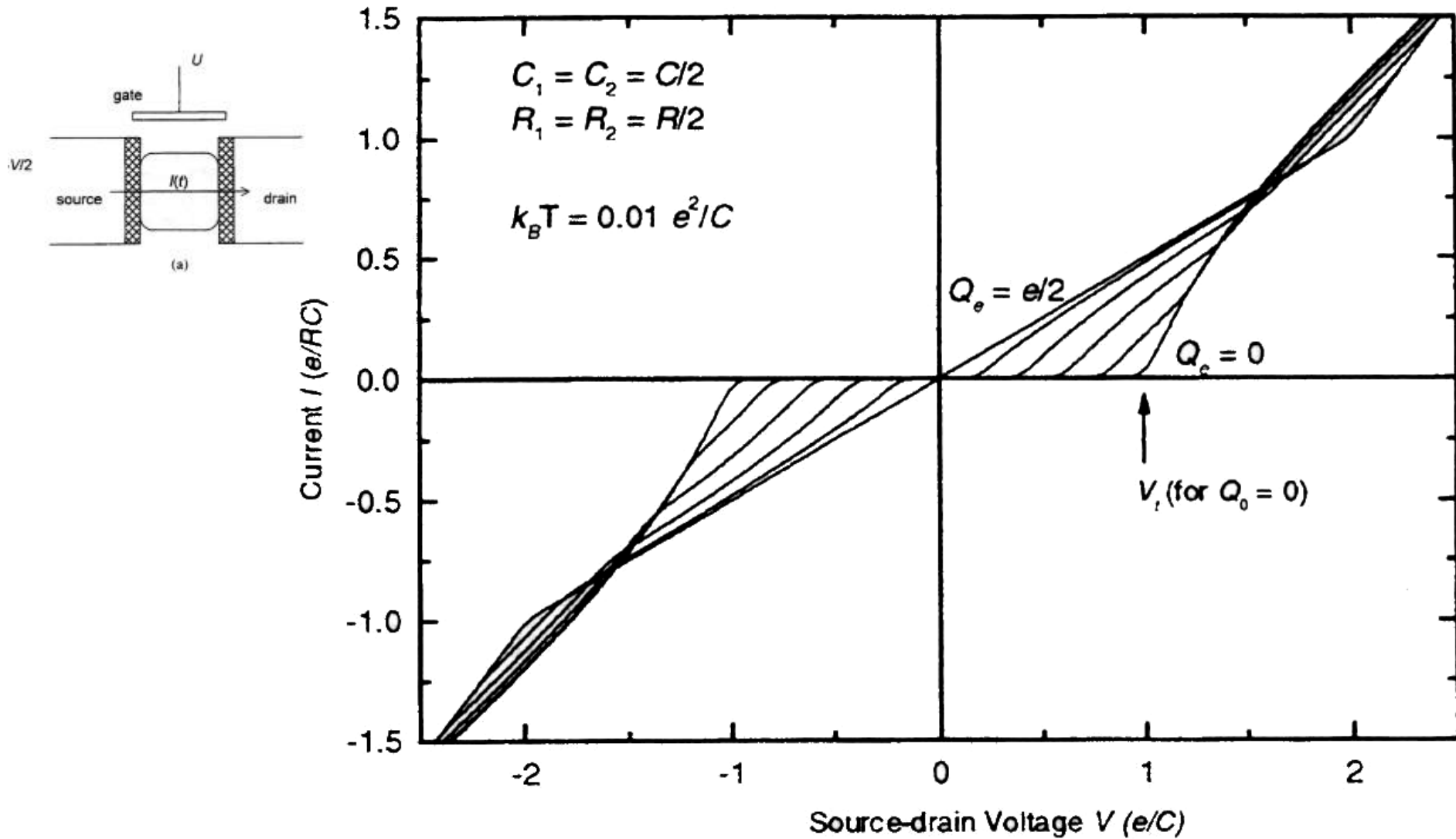
# The single electron transistor (SET)



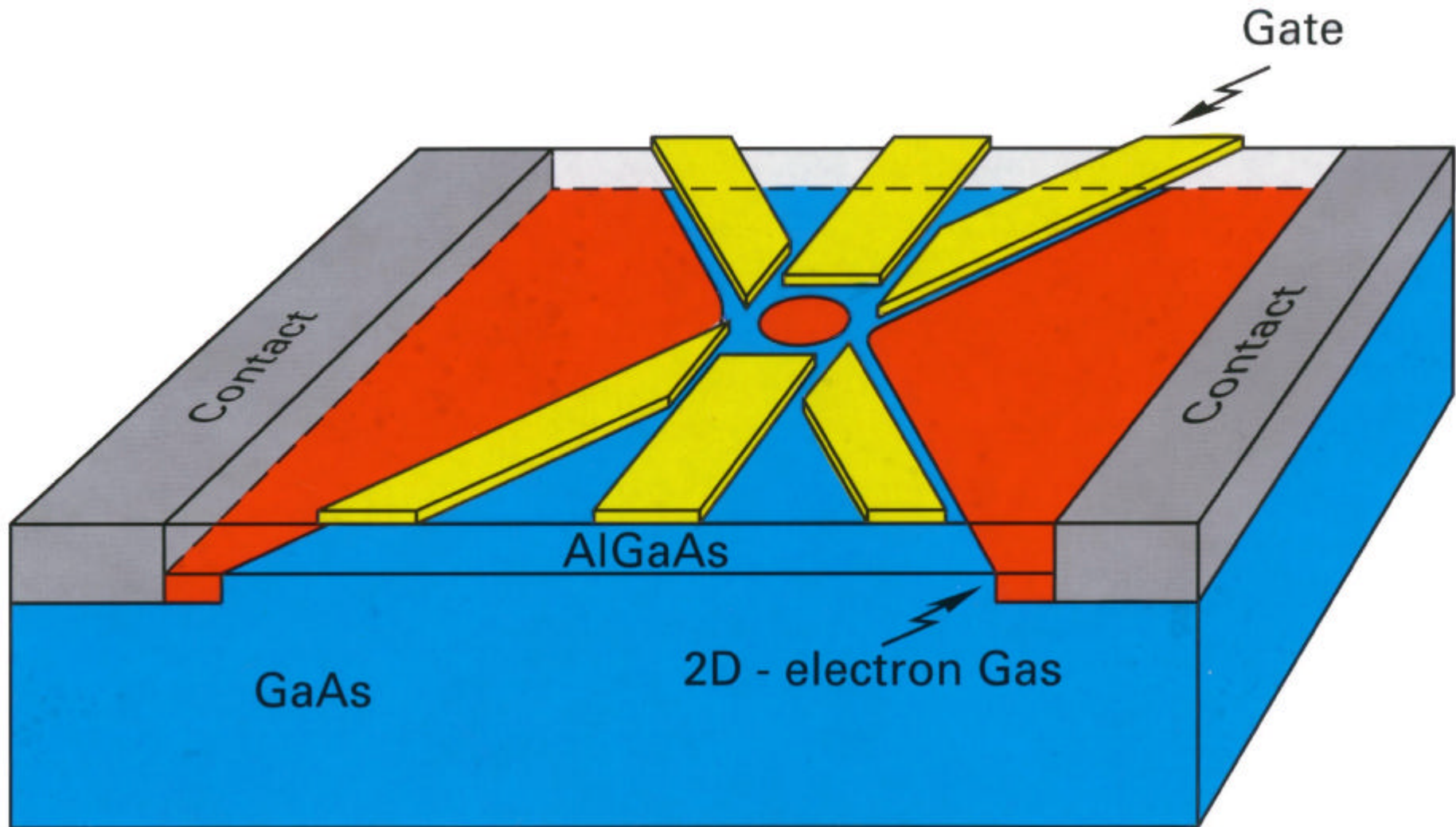
The threshold voltage  $V$  due to the Coulomb blockade oscillates as a function of the “induced charge”  $Q_e$



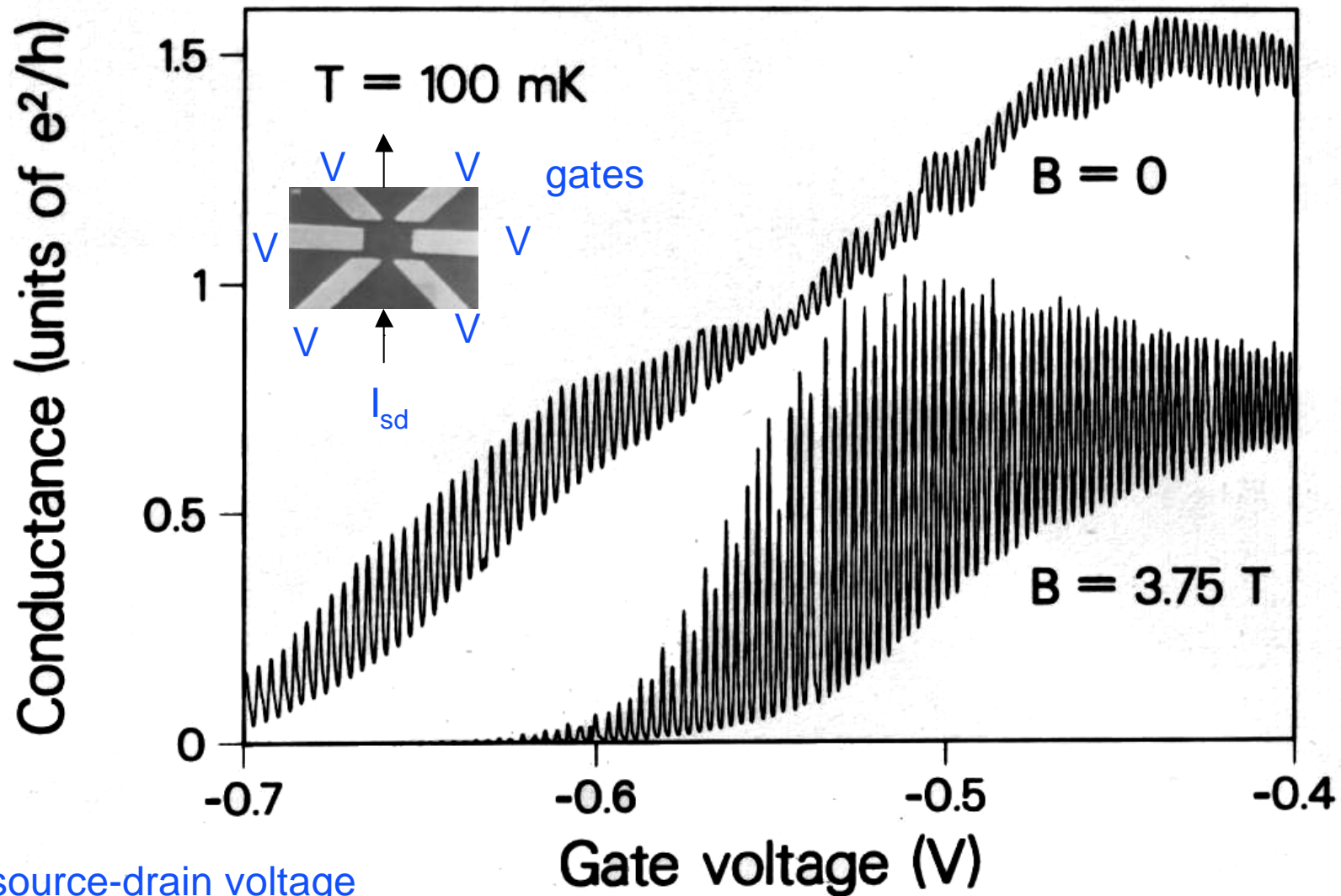
# The single electron transistor (SET)



# The quantum dot



# Coulomb blockade oscillations



Small source-drain voltage

H. van Houten, C.W.J. Beenakker, and A.A.M. Staring  
Single Charge Tunneling, 1992.

Henk van Houten, LATSIS symposium, June 2000

# The ultimate MOSFET is a single electron device...

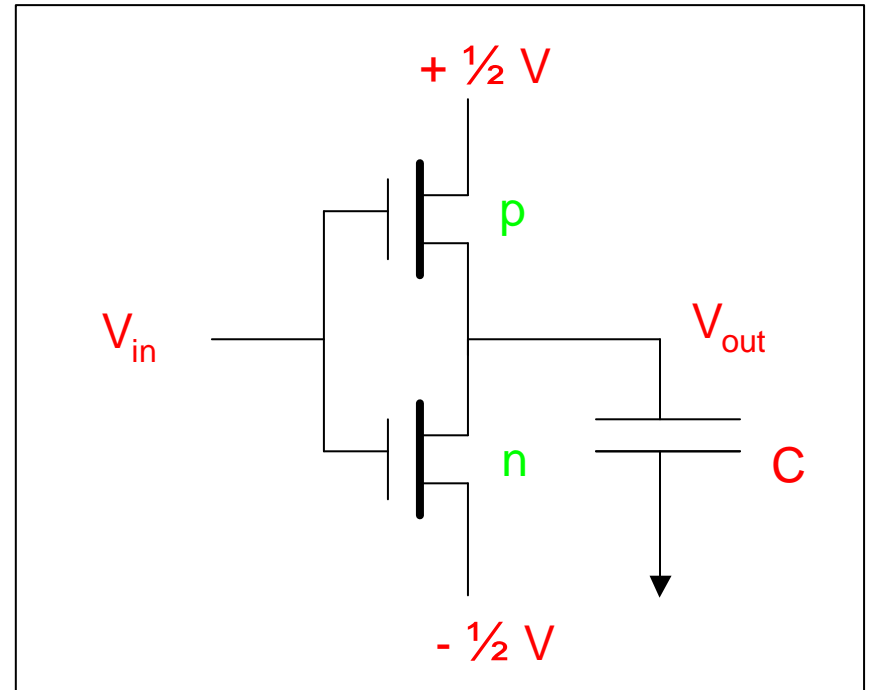
Minimum switching energy

$$E_{\text{switch}} = \frac{1}{2}CV^2 > kT$$

Gain of inverter around threshold

$$V_{\text{out}}/V_{\text{in}} \sim eV / 2kT$$

➔  $V > kT/e$  ,  $E_{\text{switch}} \sim e^2 / C$



This corresponds to the *charging energy of a single electron*

The ultimate MOSFET thus is a “single electron MOSFET”

Example:  $L = W = 10$  nm, oxide thickness 2 nm, in silicon.

It would switch in 0.1 ps (if velocity of the electron is the bulk Si saturation velocity)

# Are quantum effects relevant for the ultimate MOSFET?

- MOSFET's are used far from equilibrium
- No quantum confinement between source and drain
- charging energy typically small compared to  $kT$  at room temperature (25 meV)

Quantum effects are “washed out”

There are some effects which modify the detailed behavior

- hot electron effects
- tunneling through the gate oxide
- shifts in threshold voltage

# The quantum limit of a switching device

The Heisenberg uncertainty relation

$$\Delta E \Delta t > h / 2\pi$$

imposes a quantum limit on the power-delay product for irreversible switching

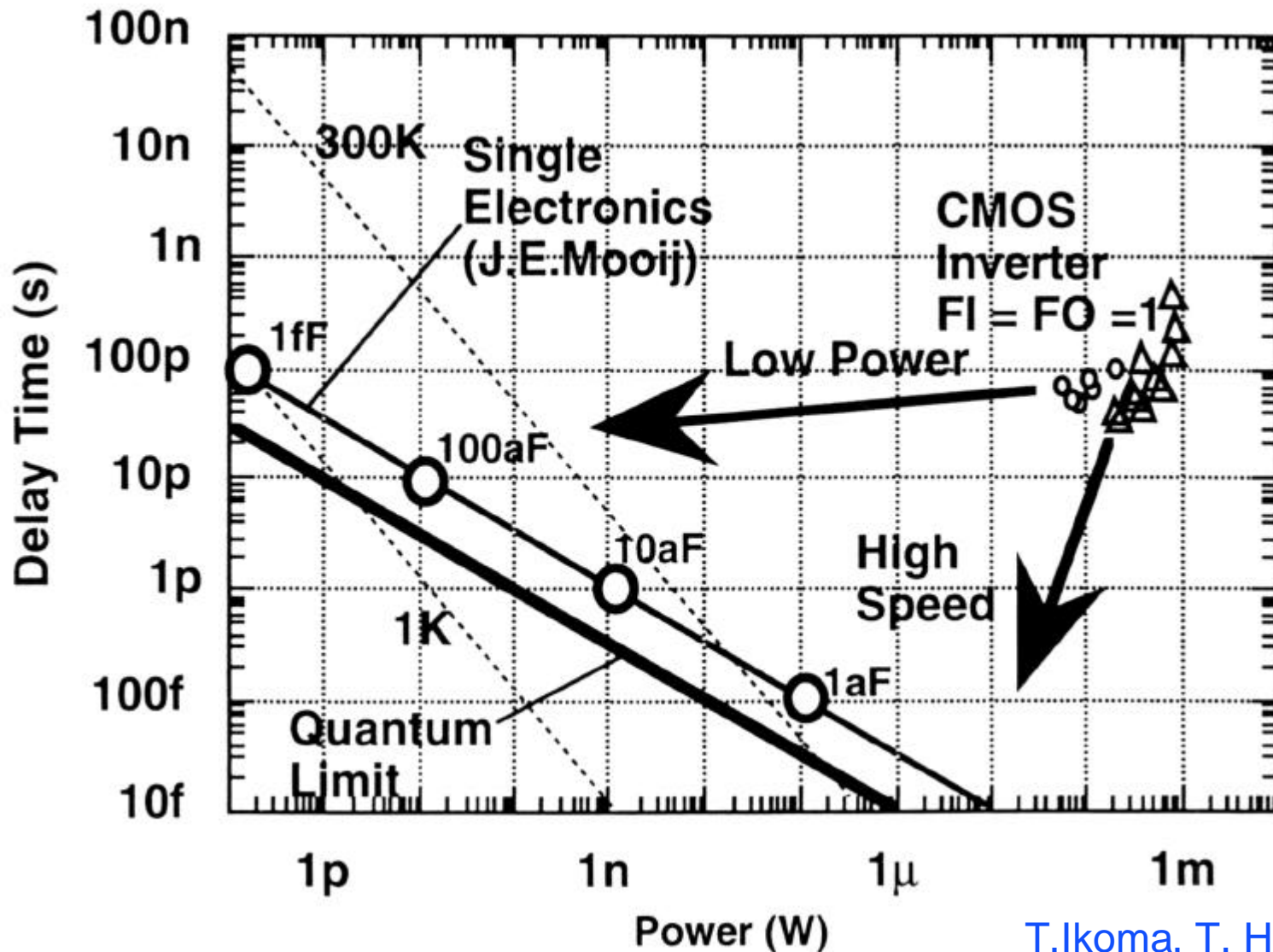
$$P \tau^2 > h / 2\pi$$

Single electronics is close to the quantum limit

$$\Delta E = e^2/C$$

$$\tau = RC \propto (h/e^2)C$$

# Quantum limit and thermal limit



Quantum limit  
 $P \tau^2 > h / 2\pi$

Thermal limit  
 $P \tau > kT$

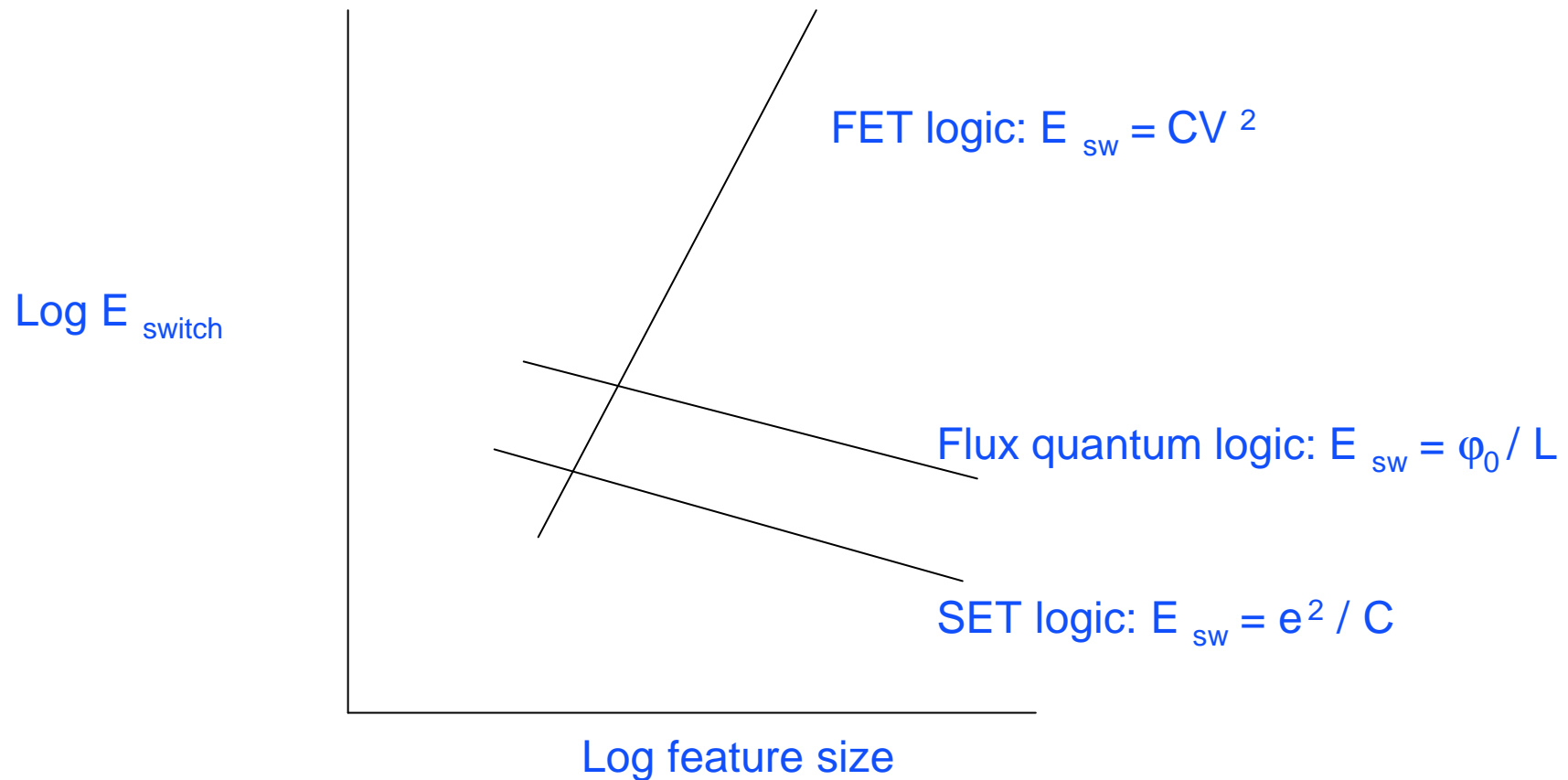
T.Ikoma, T. Hiramoto, K. Hirakawa

# Can quantum effects be used for new types of devices?

- Replacement for MOSFET highly unlikely (see Likharev)
  - MOSFET still works fine, down to SE regime
  - SET devices have typically no gain, and no logic level restoration
  - offset charges lead to unpredictable offsets
  - making identical devices is nearly impossible
  - multi-valued response is undesired for conventional architectures
  - $h/e^2 \approx 25 \text{ k } \Omega$ , poor matching to impedance of transmission line, and leading to large RC time for charging the interconnect

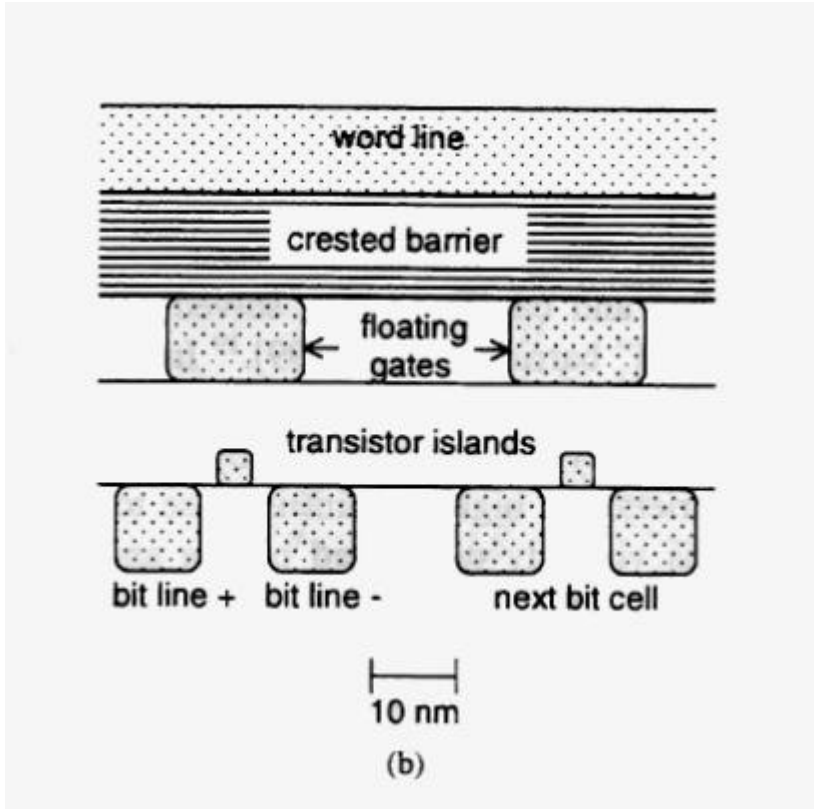
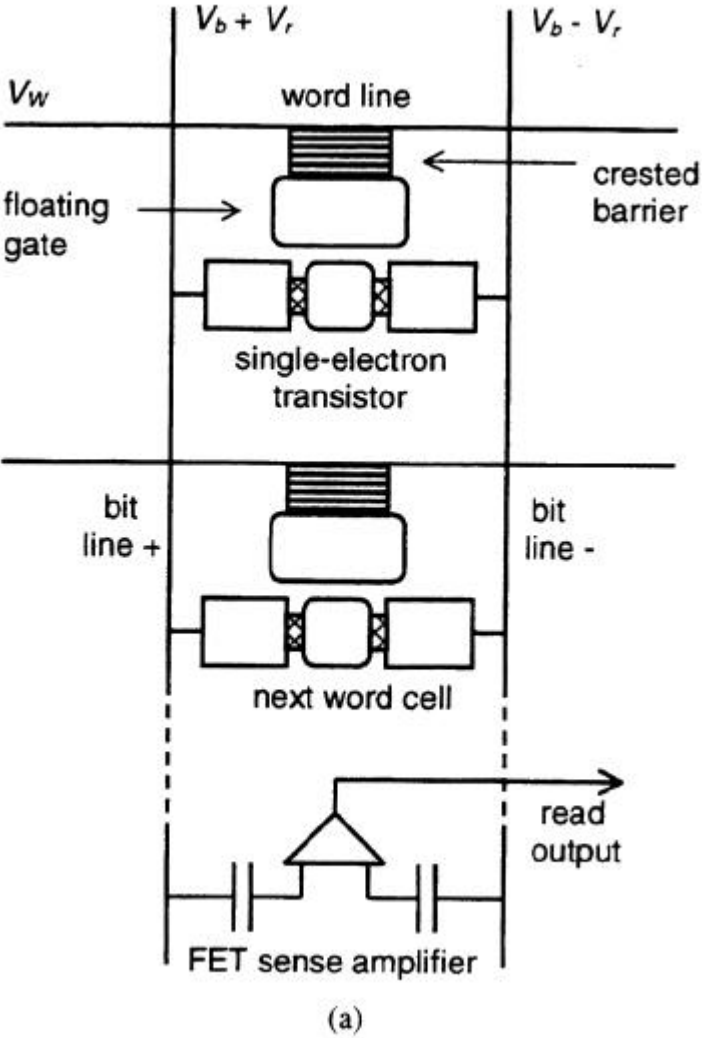


# Scaling for quantum devices



*Quantum devices dissipate more if you scale them down  
(Carver Mead and Lynn Conway, VLSI Systems)*

# SET-FET hybrid memory cell

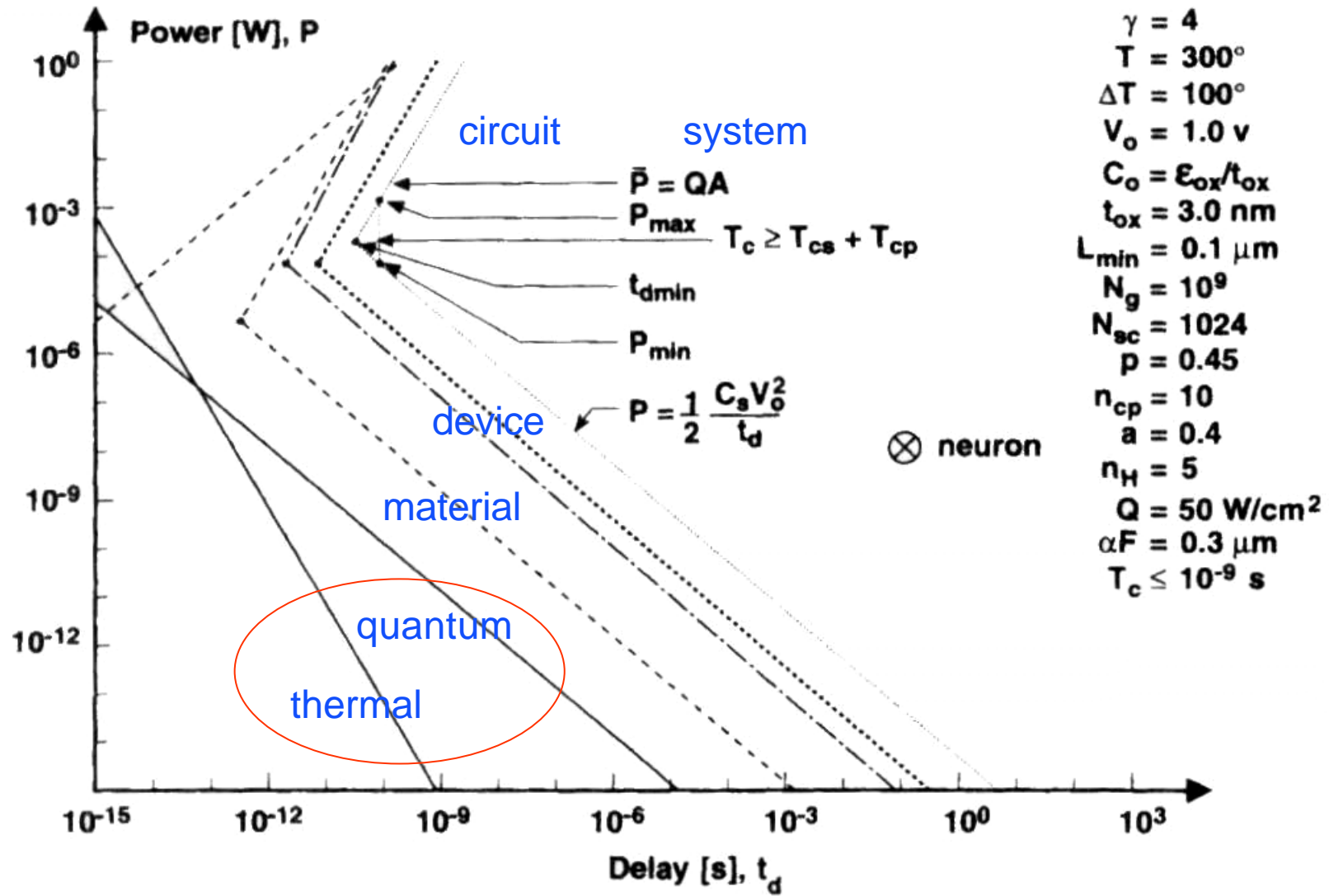


K. Likharev, Proc. IEEE, 87 (1999) 606.

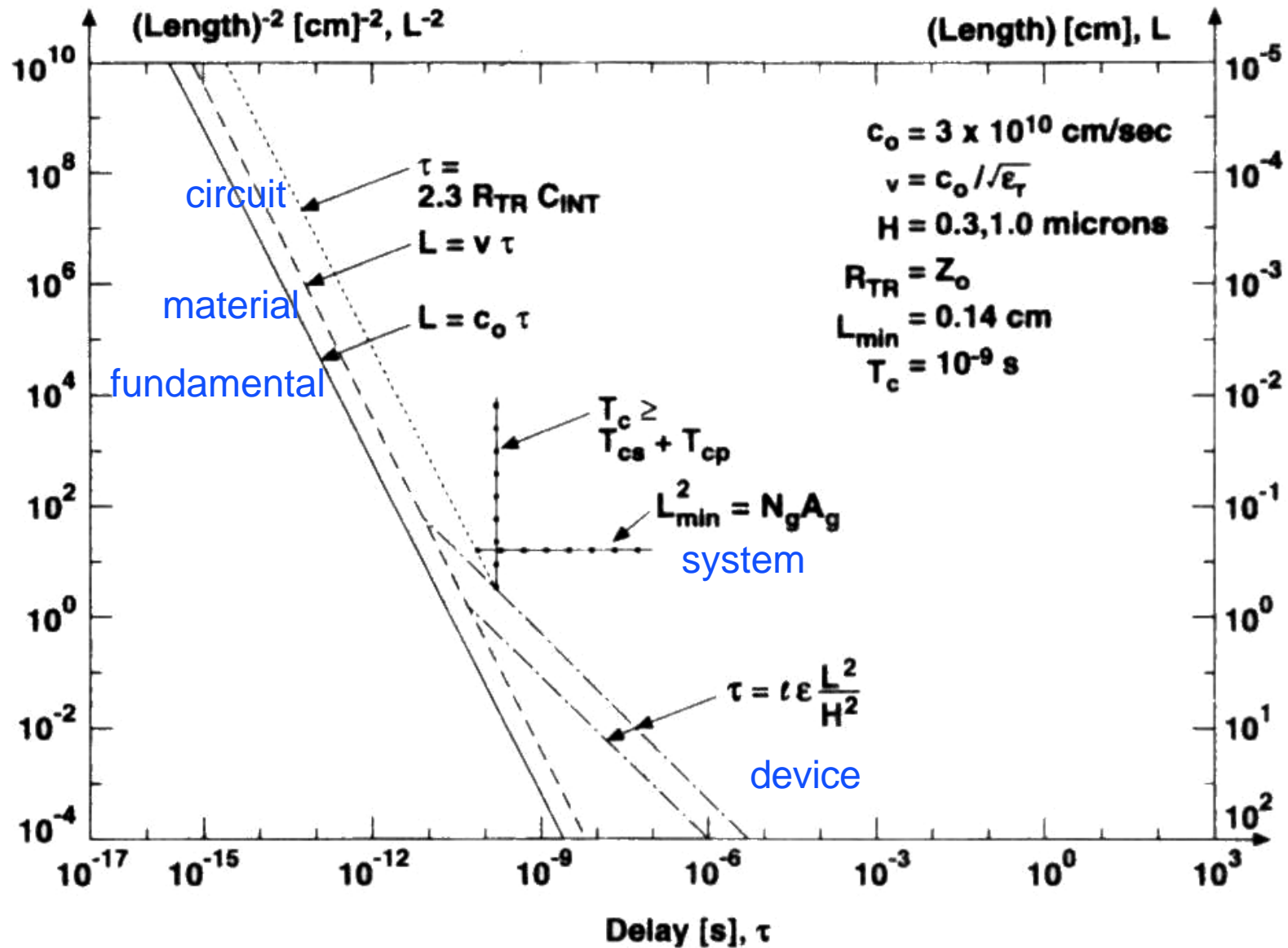
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- **A hierarchy of limits (Meindl's classification)**

# Meindl on switching limits



# Meindl on interconnect limits (case study)

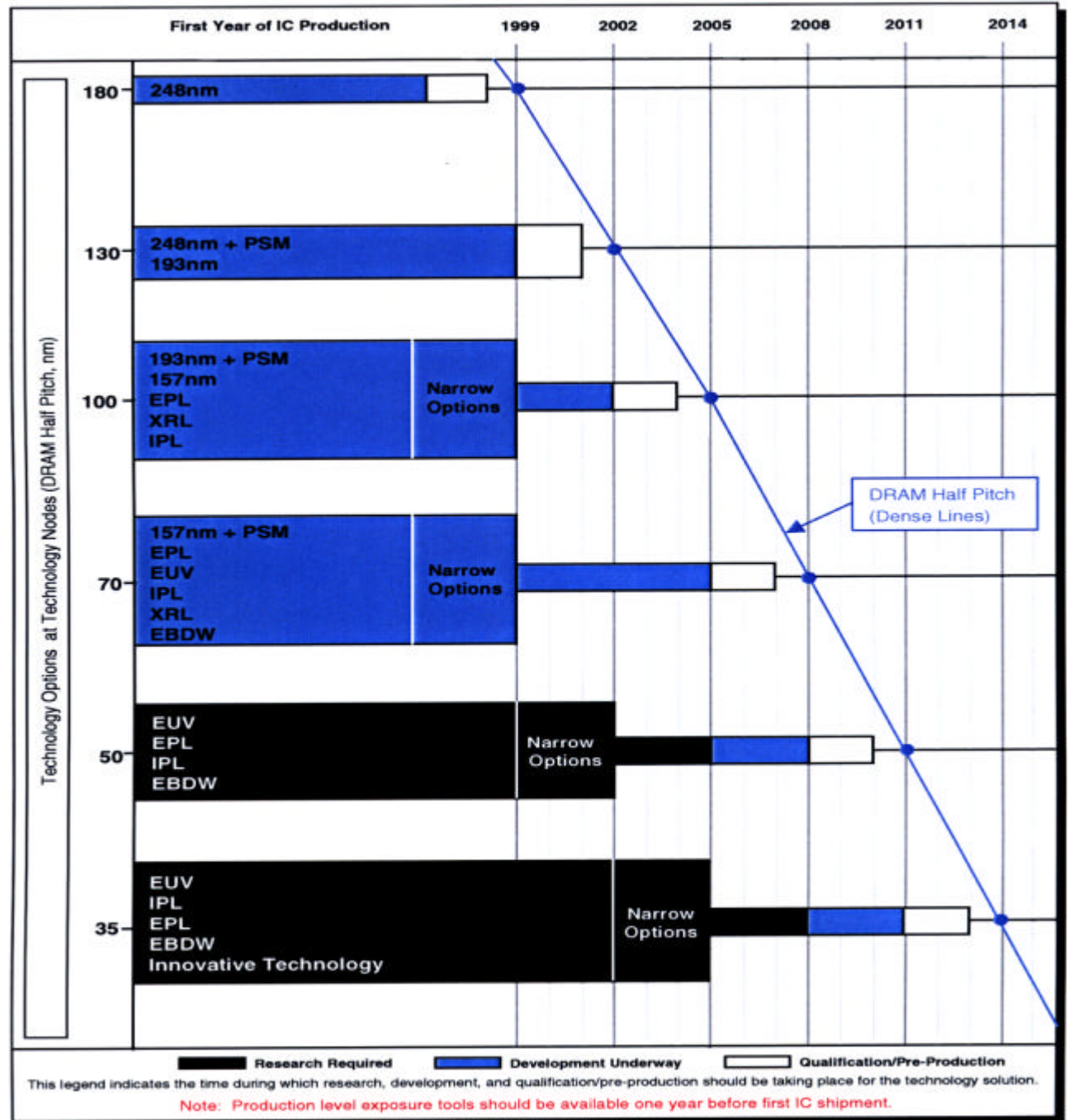


# Meindl's "hierarchy of limits"

<i>level</i>	<i>limits</i>
<b><i>System</i></b>	Ultimate system (?) 1 billion gates, 0.1 $\mu$ m CMOS, $Q = 50$ $W/cm^2$ , clock 1 ns
<b><i>Circuit</i></b>	Transfer curve, switching energy, propagation delay, global interconnect response time
<b><i>Device</i></b>	Ultimate MOSFET? $L = 50$ nm, $t_{ox} = 3$ nm $E = 0.014$ fJ = 87 eV $T < 0.5$ ps
<b><i>Material</i></b>	Saturation velocity Dielectric constant Breakdown field Thermal conductivity
<b><i>Fundamental</i></b>	Thermodynamics Quantum mechanics Electromagnetism

# Lithography (Sematech roadmap)

Two major contenders:  
EUV(13 nm) and e-projection

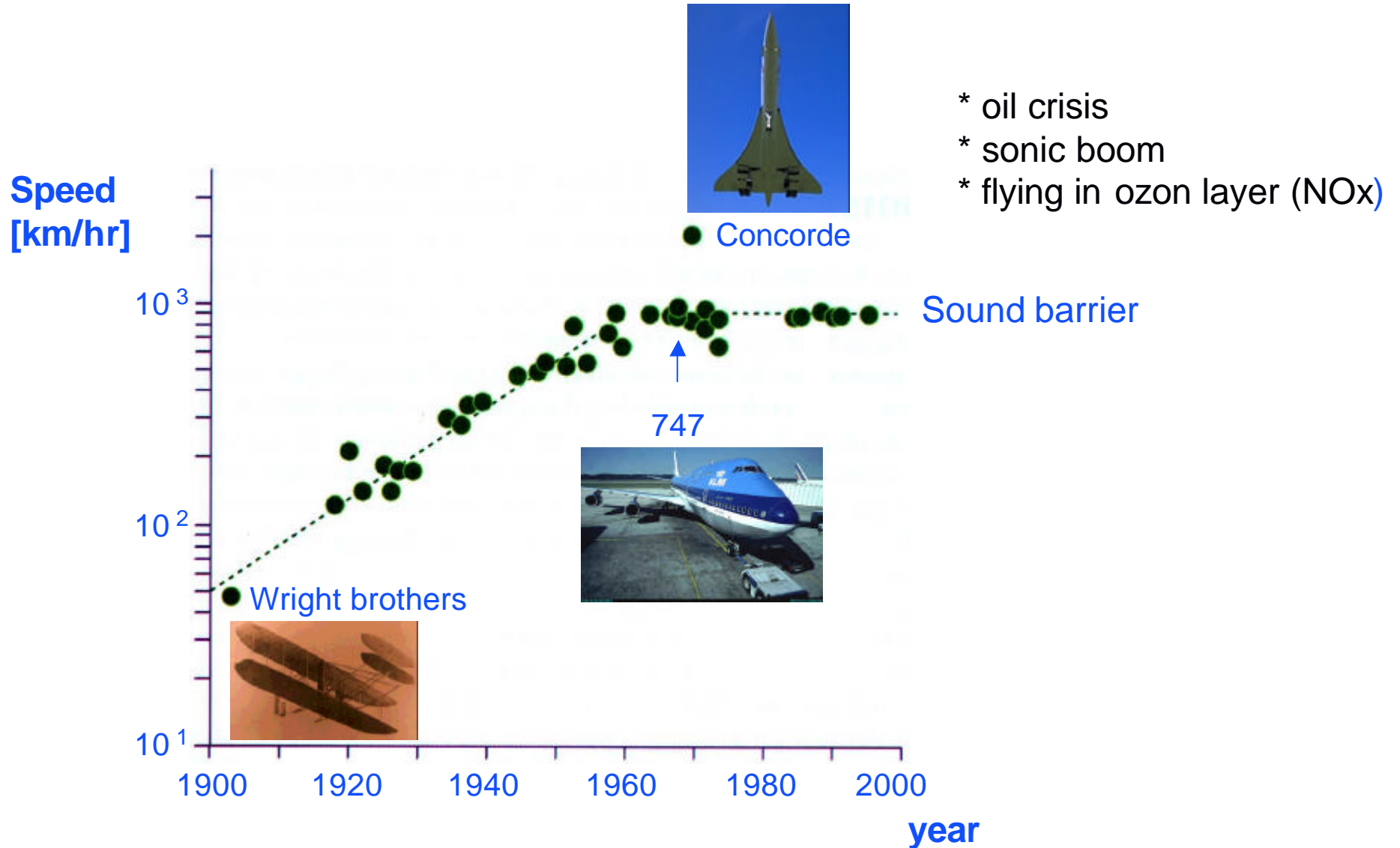


# Why Moore's law may break down (say in 2014, @ 1 Tbit DRAM)

- lithography
  - 35 nm node, 2 nm CD control for a MPU, 15 nm overlay, mask making tremendously difficult, mask and tool cost
- process technology and yield
  - gate oxide thickness <1 nm, fluctuations in doping profiles (100 atoms long gate length, 100 dopant atoms)
- power dissipation
  - high performance: heating of the chip
  - portable: battery life
- (global) interconnects
  - increasing propagation delay & parasitics
- design complexity
- economical factors



# Historic trend of aircraft speed



Source: *Nederlands Tijdschrift voor Natuurkunde* 1997

# Conclusions

- Information/computers have a physical basis
  - scaling of FET transistors is at the basis of the IT revolution
- Common wisdom physical limits are not really fundamental ...
  - Feynman 1985: “these are the only physical limitations on computers that I know of”
    - limitations to the size of atoms
    - energy requirements depending on time
    - speed of light
- ...but quantum devices seem to offer mainly disadvantages
- Practical limits and economical considerations are likely to determine how far we can stretch Moore’s law (2014?)

# Key references

- Anthony J.G. Hey, ed. *Feynman and Computation* (Perseus, Reading MA, 1999)
- Carver Mead, Lynn Conway, *Introduction to VLSI systems*, Addison Wesley, ...
- Rolf Landauer, *Dissipation and noise immunity in computation and communication*, Nature, 335, 779-784, (1988)
- James D. Meindl, *Low power microelectronics: retrospect and prospect*, Proc. IEEE, 83, 619-635 (1995); *Interconnect limits on XXI century Gigascale Integration*, Mat. Res.Soc. Symp. Proc. Vol. 514, 3-9, 1998
- Paul M. Solomon, *Critique of reversible computing and other energy saving techniques*, in *Future Trends in Microelectronics-Reflections on the Road to Nanotechnology*, ed. Serge Luryi, Jimmy Xu, and Alex Zaslavsky, NATO ASI, E 323, p. 93-109
- Carlo W.J. Beenakker and Henk van Houten, *Quantum Transport in Semiconductor Nanostructures*, Solid State Physics, 44 (1991), p. 1-228.
- Henk van Houten, Carlo W.J. Beenakker, and A.A.M. Staring, in Hermann Grabert and Michel H. Devoret, eds, *Single Charge Tunneling-Coulomb Blockade Phenomena in Nanostructures*, NATO ASI, B 294. (Plenum, New York, 1992)
- Konstantin K. Likharev, *Single electron devices and their application*, Proc. IEEE, 87, 606-632 (1999).
- Charles H. Bennett and David P. DiVincenzo, *Quantum Information and computation*, Nature, 404, 247-255, 2000