Short Course in Quantum Information Lecture 8



Physical Implementations





Course Info

• All materials downloadable @ website

http://info.phys.unm.edu/~deutschgroup/DeutschClasses.html

<u>Syllabus</u>

Lecture 1: Intro Lecture 2: Formal Structure of Quantum Mechanics Lecture 3: Entanglement Lecture 4: Qubits and Quantum Circuits Lecture 5: Algorithms Lecture 6: Decoherence and Error Correction Lecture 7: Quantum Cryptography Lecture 8: Physical Implementations





Quantum Information Processing





- 27



The DiVincenzo Criteria

Special Issue: Fortschritte der Physik, 49 (2000).

I. Scalable physical system, well characterized qubits.

- (Could be "qudits") Quantum many-body system.
- II. Ability to initialize the state of the qubits.
 - Usually a pure state on *n*-qubits $|0\rangle^{\otimes n} = |0,0,...,0\rangle$.
- III. Long relevant coherence times.
 - Much longer than gate operations (fault-tolerance).
- IV. "Universal" set of quantum gates.
 - Quantum control on the 2ⁿ dimensional Hilbert Space.
- V. Qubit-specific measurement capability. - Readout.





Example: Rydberg atom

http://gomez.physics.lsa.umich.edu/~phil/qcomp.html

Data register: Rydberg wave packet







Quantum computing in a single atom

Characteristic scales are set by "atomic units"

LengthMomentumActionEnergy
$$r_c = \frac{\hbar^2}{me^2} = a_0$$
 $p_c = \frac{me^2}{\hbar} = \frac{\hbar}{a_0}$ $L_c = r_c p_c = \hbar$ $E_c = \frac{e^2}{a_0} = \frac{p_c^2}{m}$ Bohr $L_n = n\hbar$ $r_n = n^2 a_0$ $p_n = \frac{1}{n} \frac{\hbar}{a_0}$ $E_n = -\frac{1}{2n^2} \frac{e^2}{a_0}$

Hilbert-space dimension up to *n*

$$2^{N} = \sum_{k=1}^{n} \sum_{l=0}^{k-1} (2l+1) \sim \frac{1}{3}n^{3} \sim \left(\frac{L_{n}}{\hbar}\right)^{3} = \left(\frac{r_{n}p_{n}}{\hbar}\right)^{3} + 3 \text{ degrees of freedom}$$

I. H. Deutsch, *University of New Mexico* Short Course in Quantum Information

-



Quantum computing in a single atom

Characteristic scales are set by "atomic units"

LengthMomentumActionEnergy $r_c = \frac{\hbar^2}{me^2} = a_0$ $p_c = \frac{me^2}{\hbar} = \frac{\hbar}{a_0}$ $L_c = r_c p_c = \hbar$ $E_c = \frac{e^2}{a_0} = \frac{p_c^2}{m}$ Bohr $L_n = n\hbar$ $r_n = n^2 a_0$ $p_n = \frac{1}{n} \frac{\hbar}{a_0}$ $E_n = -\frac{1}{2n^2} \frac{e^2}{a_0}$

Poor scaling in this *unary* quantum computer

$$r_n \sim 2^{2N/3} a_0$$

5 times the diameter of the Sun

 $N = 100 \text{ qubits} \implies r_n \sim 10^{20} a_0 = 6 \times 10^6 \text{ km}$

-



Hilbert space and physical resources

The primary resource for quantum computation is Hilbert-space dimension.

Hilbert-space dimension is a *physical quantity* that costs *physical resources*.





-



Hilbert space and physical resources



 $(\mathsf{B})^{|0\rangle=|000\rangle} |1\rangle=|001\rangle |2\rangle=|010\rangle |3\rangle=|011\rangle |4\rangle=|100\rangle |5\rangle=|101\rangle |6\rangle=|110\rangle |7\rangle=|111\rangle \xrightarrow{} \mathcal{X}, p$





Important Lessons

- The dimension of Hilbert is a *resource*.
- *Physics* determines the structure of Hilbert space.

• Systems with *multiple physical degrees of freedom* give Hilbert space a *tensor-product structure*.

• Control of a *many-body system* is a necessary *condition* to have an exponentially large Hilbert space without using an exponential physical resource.

R. Blume-Kohout, C. M. Caves, and I. H. Deutsch, Found. Phys. **32**, 1641(2002).





QIP = Many-body Control

n-body Hilbert Space

 $\mathcal{H} = (h_1) \otimes h_2 \otimes \cdots \otimes h_n \quad \text{(dimension 2^n)}$ "subsystem" = "body" (qubit 2^n)

Fundamental Theorem of QIP

An *arbitrary* unitary map on \mathcal{H} can be constructed from a tensor product of:

- A finite set of single-body unitaries . $\{u_i^{(1)}\}$
- Any chosen *entangling* two-body unitary. $u_{ij}^{(2)} \neq u_i^{(1)} \otimes u_j^{(1)}$





Generating Single Qubit Rotations

Rabi oscillations -- Two-level quantum dynamics:







Generating Single Qubit Rotations

Rabi oscillations -- Two-level quantum dynamics:



Apply *coherent* ac-field that couples

Field acts to "torque" the Bloch vector

Ω **Bloch Sphere.** ω_{c} In Rotating Frame. \mathcal{W}_{01}

Resonance: $\omega_c = \omega_{01}$

Rotation on Bloch sphere $\Omega \sim \langle 0|d|1\rangle E_c$



|0
angle

 $|1\rangle$



Generating Single Qubit Rotations

Rabi oscillations -- Two-level quantum dynamics:



Field acts to "torque" the Bloch vector

Bloch Sphere. In Rotating Frame.



Off-Resonance: $\Delta = \omega_c - \omega_{01}$

Rotation on Bloch sphere

$$\tilde{\Omega} = \sqrt{\Omega^2 + \Delta^2}$$





General Qubit Rotations

Hamiltonian in rotating frame:
$$\hat{H} = -\frac{\hbar\Delta}{2}\hat{Z} + \frac{\hbar\Omega}{2}\left(\cos\phi \hat{X} + \sin\phi \hat{Y}\right)$$

Rotation matrix:

$$\hat{U}(t) = \exp\left(-i\hat{H}t/\hbar\right) = \cos\frac{\Theta}{2}\hat{I} - i\sin\frac{\Theta}{2}\left(-\frac{\Delta}{\tilde{\Omega}}\hat{Z} + \frac{\Omega}{\tilde{\Omega}}(\cos\phi\,\hat{X} + \sin\phi\,\hat{Y})\right) \qquad \Theta = \tilde{\Omega}t$$

Rabi solution:

$$|\psi(0)\rangle = |1\rangle$$
 $P_0(t) = |\langle 0|\hat{U}(t)|1\rangle|^2 = \left(\frac{1-\cos(\tilde{\Omega}t)}{2}\right)\frac{\Omega^2}{\Omega^2 + \Delta^2}$



-

On resonance and $\phi = 0$: $\hat{U}(t) = \cos \frac{\Theta}{2} \hat{I} - i \sin \frac{\Theta}{2} \hat{X}$ $\hat{U}(t = \pi/\Omega) = -i \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\hat{U}(t = 2\pi/\Omega) = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

I. H. Deutsch, *University of New Mexico* Short Course in Quantum Information

 $\Delta = 0$

 $\Delta = \Omega$

 $\Lambda = 2\Omega$



Two-Qubit Entangling Gates

Canonical Example: CNOT



 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{vmatrix} 00 \\ |01 \rangle \\ |10 \rangle \\ |11 \rangle$

Example: CPhase





-

SQRT of SWAP

[1	0	0	0	$ 00\rangle$
0	$\frac{1+i}{\sqrt{2}}$	$\frac{1-i}{\sqrt{2}}$	0	$ 01\rangle$
0	$\frac{1-i}{\sqrt{2}}$	$\frac{1+i}{\sqrt{2}}$	0	$ 10\rangle$
0	0	0	1	$ 11\rangle$



Designing CPHASE Gates

Two-Qubit Separable Hamiltonian: $\hat{H}_{AB} = \hat{H}_A + \hat{H}_B \Rightarrow \hat{U}_{AB} = \hat{U}_A \otimes \hat{U}_B$

Suppose we have an interaction between qubits such that Hamiltonian is diagonal in the logical basis.

$$\hat{H}_{AB} = \begin{bmatrix} E_{00} & & & \\ & E_{01} & & \\ & & E_{10} & \\ & & & E_{11} \end{bmatrix} \quad \hat{U}_{AB} = \begin{bmatrix} e^{-i\phi_{00}} & & & \\ & e^{-i\phi_{10}} & & \\ & & & e^{-i\phi_{10}} & \\ & & & e^{-i\phi_{11}} \end{bmatrix} \quad \phi_{ij} = E_{ij}t/\hbar$$

Equivalent to CPhase by single-qubit rotations for when: $\phi_{11} + \phi_{00} - (\phi_{10} - \phi_{01}) = \pi \implies t = \frac{\pi \hbar}{E_{11} + E_{00} - (E_{10} - E_{01})}$

Requires interaction between qubits (inseparable) $\phi_{ii} \neq \phi_i + \phi_i$



-



The Tao of Quantum Computing

Coupling qubits.
Control fields.
Coherence

- Coupling to environment.
- Coupling to neglected degrees of freedom.

Decoherence





A Quantum Information Science and Technology Roadmap

Part 1: Quantum Computation

Report of the Quantum Information Science and Technology Experts Panel

"... it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway." Richard P. Feynman (1985)

Disclaimer:

The opinions expressed in this document are those of the Technology Experts Panel members and are subject to change. They should not to be taken to indicate in any way an official position of U.S. Government sponsors of this research.

> April 2, 2004 Version 2.0



Technology Experts Panel (TEP) Membership:

Chair: Dr. Richard Hughes - Los Alamos National Laboratory Deputy Chair: Dr. Gary Doolen - Los Alamos National Laboratory Prof. David Awschalom - University of California: Santa Barbara Prof. Carlton Caves - University of New Mexico Prof. Michael Chapman - Georgia Tech Prof. Robert Clark - University of New South Wales Prof. David Cory - Massachusetts Institute of Technology Dr. David DiVincenzo - IBM: Thomas J. Watson Research Center Prof. Artur Ekert - Cambridge University Prof. P. Chris Hammel - Ohio State University Prof. Paul Kwiat - University of Illinois: Urbana-Champaign Prof. Seth Lloyd - Massachusetts Institute of Technology Prof. Gerard Milburn - University of Queensland Prof. Terry Orlando - Massachusetts Institute of Technology Prof. Duncan Steel - University of Michigan Prof. Umesh Vazirani - University of California: Berkeley Prof. K. Birgitta Whaley - University of California: Berkeley Dr. David Wineland - National Institute of Standards and Technology: Boulder

http://qist.lanl.gov/





A Variety of Platforms

Atomic-Molecular-Optical

- Ion traps.
- Neutral atom traps.
- Linear optics.

Solid-State

- Semiconducting quantum dots:
 - Excitons-electronics.
 - Spintronics.
- Superconducting:
 - Cooper-pair boxes.
 - Mesoscopic circuits.

Cross-Cutting Systems

- NMR
- Canonical example: Molecules in solvents.
- Cavity QED
 - Canonical example: Single atom in a Fabry-Perot.





Carriers of Quantum Information

Electron Spin / Nuclear Spins

- Nuclear spins in molecular NMR.
- Spintronics.
- Atoms ground state electronic manifold.

Electronic motion

- Metastable excited states of atoms.
- Excitons in quantum dots.
- Macroscopic coherent current in a superconductor.

Photonics

- Polarization state of photon.
- Frequency/time encoding.
- Spatial mode.





Table 4.0-1 The Mid-Level Quantum Computation Roadmap: Promise Criteria

	The DiVincenzo Criteria						
00 Augustak	Quantum Computation					QC Networkability	
QC Approach	#1	#2	#3	#4	#5	#6	#7
NMR	Ô	8	8	\otimes	6	Ô	Ô
Trapped Ion	8	\diamond	8	\otimes	\odot	6	8
Neutral Atom	6	\diamond	0	0	6	6	6
Cavity QED	8	0	0	0		6	0
Optical	6	0	\bigotimes	8	6	8	\bigotimes
Solid State	8	0	8	8	6	Ô	Ô
Superconducting	8	\bigotimes	8	0	0	Ô	٢
Unique Qubits	This field is so diverse that it is not feasible to label the criteria with "Promise" symbols.						

Legend: 😔 = a potentially viable approach has achieved sufficient proof of principle

🚱 = a potentially viable approach has been proposed, but there has not been sufficient proof of principle

💼 = no viable approach is known

The column numbers correspond to the following QC criteria:

- #1. A scalable physical system with well-characterized qubits.
- #2. The ability to initialize the state of the qubits to a simple fiducial state.
- #3. Long (relative) decoherence times, much longer than the gate-operation time.
- #4. A universal set of quantum gates.
- #5. A qubit-specific measurement capability.
- #6. The ability to interconvert stationary and flying qubits.
- #7. The ability to faithfully transmit flying qubits between specified locations.





Ion traps

Ernshaw's Theorem

$$\nabla \cdot \mathbf{E} = 0 \Leftrightarrow \nabla^2 V = 0$$



Secular motion in oscillating trap







Paul Trap





Typical numbers

-

Dimensions: $r_0 \sim 10 \ \mu m - 1 \ cm$ Voltage: $\tilde{U} \sim 100 - 500V$, $|U| \sim 0 - 50V$ rf-frequency: $\omega_{rf} \sim 100 \ kHz - 100 \ MHz$ Secular frequencies: $\omega_{rf} \sim 10 \ kHz - 10 \ MHz$



Normal modes of motion



"center of mass"

$$\omega_{cm} = \omega_0$$



"stretch mode"

$$\omega_{st} = \sqrt{3}\omega_0$$





Encoding quantum information

Internal states



Readout



"Cycling transition": Many photons scattered.

Ion "lights up" when in |0
angle





Quantized Motion

"Resolved Sidebands" - Quantized Doppler Effect.

Both internal and external degrees of freedom quantized.

$$\Psi \rangle = |\phi_{\text{int}}\rangle \otimes |\chi_{ext}\rangle \qquad |\phi_{\text{int}}\rangle = \{|e\rangle \text{ or } |g\rangle\} \qquad |\chi_{ext}\rangle = \{|n\rangle\}$$



-



Quantized Motion

"Resolved Sidebands" - Quantized Doppler Effect.

Both internal and external degrees of freedom quantized.

$$\Psi \rangle = |\phi_{\text{int}}\rangle \otimes |\chi_{ext}\rangle \qquad |\phi_{\text{int}}\rangle = \{|e\rangle \text{ or } |g\rangle\} \qquad |\chi_{ext}\rangle = \{|n\rangle\}$$



-



Quantized Motion

"Resolved Sidebands" - Quantized Doppler Effect.

Both internal and external degrees of freedom quantized.

$$\Psi \rangle = |\phi_{\text{int}}\rangle \otimes |\chi_{ext}\rangle \qquad |\phi_{\text{int}}\rangle = \{|e\rangle \text{ or } |g\rangle\} \qquad |\chi_{ext}\rangle = \{|n\rangle\}$$



-



Rabi Oscillations



I. H. Deutsch, *University of New Mexico* Short Course in Quantum Information



Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria (Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.



- Ions in a linear Paul trap.
- Use quantized normal modes as a "quantum bus".
- Resolved sidebands couple internal-external qubits.





Two-qubit gate: Cirac-Zoller I

Atomic qubit $\{|g\rangle, |e\rangle\}$ Motional qubit $\{|0\rangle, |1\rangle\}$

Employ "auxiliary" level $|aux\rangle$. Apply a 2π rotation on the blue-sideband of $|g\rangle \leftrightarrow |aux\rangle$



Achieves C-Phase between internal and external qubit

$$\begin{aligned} |g\rangle|0\rangle &\to |g\rangle|0\rangle \\ |e\rangle|0\rangle &\to |e\rangle|0\rangle \\ |g\rangle|1\rangle &\to -|g\rangle|1\rangle \\ |e\rangle|1\rangle &\to |e\rangle|1\rangle \end{aligned}$$





Two-qubit gate: Cirac-Zoller II

Motional qubit as "quantum bus": Oscillation mode shared.

• Swap control-qubit with motional qubit







Two-qubit gate: Cirac-Zoller II

Motional qubit as "quantum bus": Oscillation mode shared.

• C-Phase between motional qubit and target







Two-qubit gate: Cirac-Zoller II

Motional qubit as "quantum bus": Oscillation mode shared.

Swap motional qubit back with control









Experiments on Cirac-Zoller Gate

• 1995 Monroe et al.: Entangling motional and internal qubit.



Q-circuit = Ramsey Interferometer



Ramsey detuning (kHz)

• 2003 Schmidt-Kaler et al.: Multiqubit entanglement







Early Issues with Ion Trap QC

Anomalous Heating

2.5

1.5

0.53

2

- Fluctuating patch potentials.
- Poor scaling with trap size.
- Want large oscillation freq.











0 Ion Position (normalized)

- 100 1

2





Scaling Up: Multiplex Segmented Trap



D. Kielpinski, C. Monroe, and D. J. Wineland, Nature 417, 709 (2002). To appear in the 2005 International Symposium on Microarchitecture (MICRO-38)

A Quantum Logic Array Microarchitecture: Scalable Quantum Data Movement and Computation

Tzvetan S. Metodi[†], Darshan D. Thaker[†], Andrew W. Cross[‡] Frederic T. Chong[§] and Isaac L. Chuang[‡]

Operation	Time	Pcurrent	Pexpected
Single Gate	1 <i>µ</i> s	0.0001	10 ⁻⁸
Double Gate	10µs	0.03	10 ⁻⁷
Measure	100µs	0.01	10 ⁻⁸
Movement	10 <i>ns/µ</i> m	0.005/µm	10 ⁻⁶ /cell
Split	10µs	0.50	28
Cooling	1μ s		
Memory time	10-100 sec		



Qubits in Quantum Dots



Coherent Manipulation of Coupled Electron Spins in Semiconductor Quantum Dots

J. R. Petta,¹ A. C. Johnson,¹ J. M. Taylor,¹ E. A. Laird,¹ A. Yacoby,² M. D. Lukin,¹ C. M. Marcus,¹ M. P. Hanson,³ A. C. Gossard³

Science 309, 2184 (2005).







Qubits in Josephson Junctions



Coherent Quantum Dynamics of a Superconducting Flux Qubit

I. Chiorescu, ^{1*} Y. Nakamura, ^{1,2} C. J. P. M. Harmans, ¹ J. E. Mooij¹

Science 299, 189 (2003).







Qubits in Neutral Atoms



Controlled collisions for multiparticle entanglement of optically trapped atoms

Olaf Mandel, Markus Greiner, Artur Widera, Tim Rom, Theodor W. Hänsch & Immanuel Bloch

Nature 425, 937 (2003).



I. H. Deutsch, *University of New Mexico* Short Course in Quantum Information



Time $\pi/2$ $\pi/2$ j+1 j+1 1+2 1+2 Lattice site Lattice site а 0,8 φ=0 Rel. atom number 0,6 Ramsey 0.4 Fringes 0,2 0 100 200 300 Phase, a (degrees) С

 $\pi/2$

 $\pi/2$



Qubits in Photons



A scheme for efficient quantum computation with linear optics

E. Knill*, R. Laflamme* & G. J. Milburn†

Nature 409, 46 (2001)

Demonstration of an all-optical quantum controlled-NOT gate

J. L. O'Brien^{1*}, G. J. Pryde^{1*}, A. G. White¹, T. C. Ralph¹ & D. Branning^{1,2}

Nature 426, 264 (2003)





Conclusions

• Scalable quantum computing requires coherent control of a *many-body* system.

- Arbitrary control through single-body (qubit) unitaries operations plus two-body entangling operations.
- Must overcome the intrinsic conflict: Require strong coupling between qubits and to controlling fields.
- Variety of platforms from single photons to macroscopic persistent current.
- Experiments are proceeding to the point where scaling up to many qubits requires an engineering solution.



