Gait Analysis for Six-Legged Robots

Josep M. Porta and Enric Celaya
Institut de Robòtica i Informàtica Industrial (UPC-CSIC)
Gran Capità 2-4, 08034, Barcelona, SPAIN
e-mail: jporta@iri.upc.es, ecelaya@iri.upc.es

Abstract

We present a general method for the analysis of the gaits used by a six-legged robot independently of the mechanism used to generate the gait. The gait state of the robot is defined as a function of the last executed steps and several classes of gait states as well as the transitions between them are identified. As an example, we apply our method to the well-know wave gaits (the most efficient and stable gaits for straight line locomotion on flat terrain) showing interesting properties about how they can be generated, the relation between all the possible wave gaits and about the emergence of the tripod gait under certain assumptions likely to hold when walking on flat terrain.

1 Introduction

In the last years there has been a growing interest in the area of leggeds robots. This increasing interest is justified by the fact that a legged robot is able to deal with very irregular terrain in which wheeled robots are unable to move. Since the main part of the outdoor environments are constituted by abrupt terrain, legged robots represent an opportunity to enlarge the application area of the autonomous robots.

One of the aspects related with the control of legged robots that has received more attention is that of the generation of statically stable gaits. The task of a gait generation mechanism can be defined as selecting an appropriate sequence of leg and body movements so that the robot advances with a desired speed and direction.

In the literature of legged robots there are many different approaches to the problem of building controllers for gait generation. For instance, in [6] a controller that generates periodic gaits is described while [7] is about free gait controllers and other works combine both approaches [4]. Some authors use a centralized mechanism to produce a gait [2] but others prefer distributed ones [3]. Some of the existent works in gait generation try to imitate the gaits observed in insects [5] since they are adaptive and robust, but there are also works that advocate in favor of an engineering perspective [2]. Some controllers are basically reflexive [1] departing from other approaches that are clearly deliberative [8].

The great diversity and relative complexity of these controllers make their comparative analysis difficult and their performances are evaluated in a rather subjective way.

We present a general method for the analysis of the gait of a six legged robot without using information about the logics and implementation details of its generation mechanism. This method is based on the sequence of steps issued by the controller. This sequence allows

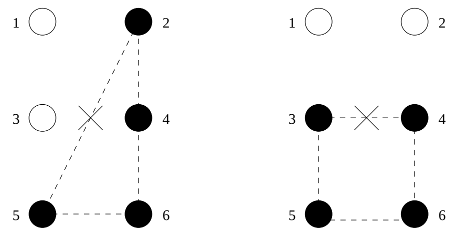

Figure 1: *Examples of the two kinds of possible, potentially unstable, situations when lifting two neighboring legs simultaneously. Black dots represent supporting legs and white ones, legs on the air. The support polygon is represented by dashed lines and the projection of the center of gravity of the robot by a cross.*

to define the gait state of the robot at every time. From this abstract point of view, aspects such as foothold search or body posture control are neglected.

In the following sections we describe our method for the analysis of a gait. First a gait state of the robot is defined as a function of few last executed steps. In section 3 the transitions between gait states in a six legged robot are classified. Then, in section 4 the methodology described in previous sections is applied to analyse the wave gaits. In section 5 some conclusions are derived.

2 The Gait State of a Legged Robot

Observing the outcome of any gait generation mechanism we can record the temporal sequence of step executions. This is what we call the *gait sequence* and is the base of our gait analysis. The gait sequence can be represented by a temporally ordered string of step starting events where the last executed step corresponds to the rightmost position of the string. When two or more steps are performed at the same time they are represented with a pair of parenthesis embracing them. As an exemple, the gait sequence:

$$\ldots, L_1, L_3, (L_4, L_5), L_6, L_2 \tag{1}$$

indicates that the six last steps of the robot have been first L_1, then L_3, then L_4 and L_5 at the same time, then L_6, and finally L_2, with the leg numbering shown in figure 1.

Since we are only interested in statically stable gaits we will exclude from analysis those gaits that do not provide a sufficient stability margin to the robot. In general, in a six legged robot, a reasonable stability is guaranteed if the following rule is always fulfilled:

Rule 1: *Two contiguous legs are never on the air at the same time.*

The violation of this rule would leave the robot in a situation like those represented in figure 1 with a very poor stability. Rule 1 implies that two contiguous legs can not start a step at the same time, and this means that using our notation for describing gait sequences, two contiguous legs can never appear together inside a pair of parenthesis. This provides a way to define a non-ambiguous *stepping relation* between contiguous legs. For any statically stable gait sequence and for any couple of contiguous legs 'a' and 'b', we will have either that 'a' has performed a step more recently than 'b' or the converse, which will be expressed as $L_a > L_b$ and $L_a < L_b$, respectively.

According to this we define the *gait state* of a legged robot as the set of the stepping relations between all the couples of neighboring legs derived from the present gait sequence.

R	Class Name	Number of Elements	Generic Gait State
1	A_1	6	$a > b < c < d < e < f < a$
	A_2	6	$a < b < c < d < e > f > a$
2	B_2	6	$a < b < c < d > e < f > a$
	C_2	3	$s < b < c > d < e < f > a$
	A_3	6	$a > b > c > d < e < f < a$
3	B_3	6	$a > b < c > d > e < f < a$
	C_3	6	$a > b > c < d > e < f < a$
	D_3	2	$a > b < c > d < e > f < a$
	A_4	6	$a > b > c > d > e < f < a$
4	B_4	6	$a > b > c > d < e > f < a$
	C_4	3	$a > b > c < d > e > f < a$
5	A_5	6	$a > b > c > d > e > f < a$

Table 1: *The 12 classes of gait states. The sequence (a,b,c,d,e,f) may by substituted by any cyclic permutation of ($L_1, L_3, L_5, L_6, L_4, L_2$)*

The gait state can be represented by a string in which contiguous legs of the robot are represented in contiguous positions with the stepping relations between them. For instance, if we follow an anti-clockwise path starting with the left front leg, then the gait state of the robot after performing the gait sequence (1) is:

$$L_1 < L_3 < L_5 < L_6 > L_4 < L_2 > L_1. \quad (2)$$

The gait state includes some relevant information that can be extracted from the gait sequence as for instance, which legs are more likely to step next (those that have stepped more time ago than their two neighbors, L_1 and L_4 in the example), or which legs have already finished their return stroke and are expected to be in contact with the ground. Thus, when a leg performs a step its two neighbors must be in support phase, and they will remain in this state until they perform their next step. So only the legs that have executed a step after their two neighbors (L_2 and L_6 in the example 2) may be in return stroke while the rest of legs must be in support phase.

For each gait state we define R as the number of legs that have stepped more recently than their right (anti-clockwise) contiguous leg or, equivalently, as the number of symbols '>' that appear in the gait state representation introduced above. R is a number between 1 and 5. In the example (2) we have R=2.

In a six legged robot, there are six couples of neighboring legs and given that there are two possible relations ('>' and '<') between each couple of legs then the total number of possible combinations is $2^6 = 64$. But, due to the transitivity of same-type temporal relations, two of these combinations are not valid (the one with six '>', and the one with six '<') so we have 62 possible gait states. These gait states can be grouped in 12 classes that include all those states that are equal except for a cyclic permutation and that are shown in table 1. This classification is useful to simplify the gait analysis since all the states within a class have the same behavior and to prove any property we only need to examine one member of each class instead of analysing the 62 possible states.

3 State Transitions

Every time that a step is performed the stepping relations between legs change modifying the gait state of the robot. For each gait state six transitions are possible (one for the step of each leg). The graph of figure 2 shows the possible transitions between classes of states if only one step execution at a time is considered. If two or three steps are executed simultaneously then they can be analysed in any desired order since the final result is the same. In that graph there is a clear symmetry around states with R=3. As we will see in the next section, states with R=3 play a central role in the gait generation problem.

Attending to the stepping relations existing before the execution of a step between the leg that performs the step and its neighboring legs, four classes of steps can be identified:

Type of Step	Previous State	Next State
Alternating	$\ldots a > b < c \ldots$	$\ldots a < b > c \ldots$
Repeated	$\ldots a < b > c \ldots$	$\ldots a < b > c \ldots$
Additive	$\ldots a < b < c \ldots$	$\ldots a < b > c \ldots$
Subtractive	$\ldots a > b > c \ldots$	$\ldots a < b > c \ldots$

An *alternating* step results when a leg that has stepped before its two neighbors is moved, resulting in an alternation between steps of contiguous legs. In general, alternating steps between contiguous legs is a good heuristic to move the more retracted legs first. This is an interesting property since executing steps with the more retracted legs reduces the number of steps required to cross a given distance. Another interesting property related with alternating steps is that periodic gaits that involves all legs only once in a cycle use exclusively alternating steps. This is obvious considering that, in such conditions between two consecutive steps of a leg all other legs have stepped once, and that includes its two neighbors.

A *repeated* step is the re-issue of a recently executed step without moving any of its neighboring legs. A repeated step does not change the gait state so they are represented as loops in the graph of figure 2. A repeated step corresponds to the execution of a short step that can be necessary for repositioning a leg.

Both alternating and repeated steps are called *conservative* steps in the sense that they keep R constant.

Additive steps increase R and correspond to transitions from one layer of figure 2 to the next lower one.

Subtractive steps produce the reverse effect since they decrease R. The transitions from the bottom to the top of the graph correspond to subtractive steps.

In general, any gait generation mechanism can be characterized by the frequency with which each of the transitions of graph in figure 2 is followed. For instance, a periodic gait controller follows a closed path in the space of states, while with a controller that generates a free gait all transitions can eventually be used.

4 Analysis of the Wave Gaits

The most efficient and stable gaits for straight motion on flat terrain are the wave gaits [9]. Those gaits are commonly observed in walking animals [10] and are the family of gaits

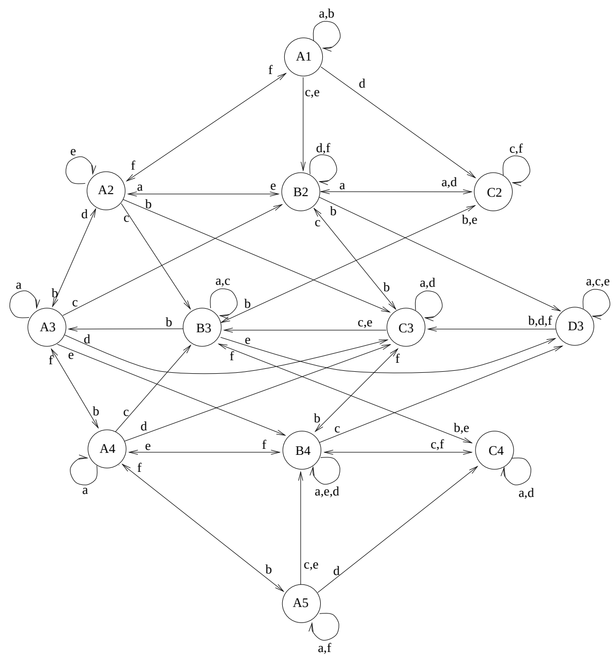

Figure 2: *Graph of transition between classes of gait states. The interpretation of the letters for each class of gait states is that of table 1*

that most of the existent controllers try to generate [1], [5], [6]. In the following sections we apply the methodology described in previous sections to the analysis of the wave gaits.

4.1 Wave Gaits Description

A wave gait is characterized by the following conditions:

$$
\begin{aligned}
t_i^n &= t_{i+2}^n + T_p \qquad 1 \le i \le 4 \\
t_6^n &= t_5^n + T_w/2
\end{aligned}
$$

with the leg numbering of figure 1, where T_w is the period of the gait, t_i^n is the starting time of the step number n of leg i and T_p is the protraction time which is constant for all steps. The parameter T_w can vary from $2T_p$ to $6T_p$ given rise to the whole family of wave gaits. So, all possible wave gaits are obtained by separating the execution of the steps of the two rear legs.

Attending to the resulting gait sequence (which is the only thing that matters for our analysis) it is possible to classify all the continuous spectrum of wave gaits in only four classes. The gait sequences corresponding to these four classes are iterations of the following fragments of gait sequences:

- Slow sequence: $L_6, L_4, L_2, L_5, L_3, L_1$
- Crossed sequence: $(L_1, L_6), L_4, (L_2, L_5), L_3$
- Ripple sequence: $L_6, L_1, L_4, L_5, L_2, L_3$
- Tripod sequence: $(L_1, L_4, L_5), (L_2, L_3, L_6)$

The crossed and tripod sequences correspond to singular cases in which some events coincide in time while the slow and ripple sequences include continuous sets of wave gaits.

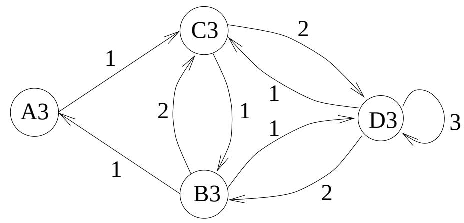

Figure 3: *Graph of transition between classes of gait states with R=3. The numbers of the arrows indicate how many legs must execute a step simultaneously to perform the corresponding transition.*

4.2 Gait States

It is possible to see that the R corresponding to all the gait states in which a robot can be while executing any wave gait is 3. To prove that we only have to demonstrate the following two points:

- *The execution of a wave gait keeps R constant.*
  This is evident taking into account that wave gaits are periodic gaits that use all legs only once in a cycle. As we have seen in section 3, this kind of gaits use only alternating steps, which keep R constant.

- *The R of some gait state of each wave gait is 3.*
  This can be seen evaluating the state of the robot at an arbitrary moment. If we do that after the execution of each one of the gait sequence fragments identified in the previous section we get:

  - Slow: $L_1 > L_3 > L_5 > L_6 < L_4 < L_2 < L_1$
  - Crossed: $L_1 < L_3 > L_5 > L_6 < L_4 < L_2 > L_1$
  - Ripple: $L_1 < L_3 > L_5 > L_6 < L_4 < L_2 > L_1$
  - Tripod: $L_1 < L_3 > L_5 < L_6 > L_4 < L_2 > L_1$

  These four states belong to the classes of gait states A_3, B_3, B_3, and D_3, respectively and all of them have R=3.

So, while a robot executes any wave gait, it is always in a gait state with R=3.

4.3 Transitions between Wave Gaits

In this section we examine which gait states are used by each kind of wave gait to find out how to change in a smooth way from one wave gait to another.

In figure 3 the possible transitions between states with R=3 are shown. This graph is built from that of figure 2 but including the possiblity of executing more than one step at a time. Since wave gaits are periodic gaits, they must pass through the same states cyclically. The paths followed by each type of wave gait can be identified studying the evolution of the gait state of the robot when the corresponding gait sequence is executed. The resulting cycles are:

1. Cycle for the slow gaits: A_3, B_3, C_3.

2. Cycle for the crossed gait: B_3, C_3.
3. Cycle for the ripple gaits: B_3, D_3, C_3.
4. Cycle for the tripod gait: D_3.

Since the graph of transitions between states with R=3 (figure 3) is densely connected, there is always one point of the previously described cycles from which the rest of cycles can be reached using only alternating steps. So, by preventing or avoiding the simultaneous execution of some steps in the correct states it is possible to pass from one type of wave gait to any other. For instance, if while performing tripod gait one of the three legs that should step together is forced to move before than the other two, the resulting gait state will be of type C_3 and from this point any other kind of wave gait (slow, crossed or ripple) can follow.

4.4 The Tripod Gait

The tripod gait deserves a special consideration since it is the fastest statically stable gait that a six legged robot can use. In the tripod gait the gait state alternates between the two states of the class D_3.

Now we are interested in the conditions under which a tripod gait will be obtained. Suppose that the protraction time is constant for all legs and that the robot executes only alternating steps as soon as possible, that is, when appropriate stepping relations between a leg and its two neighboring legs exist and inmediately after these neighboring legs reach contact with the ground. With these suppositions, which are likely to be fulfilled when the robot walks on flat terrain, we want to demonstrate that independently of the initial position of the legs, if the robot is in a D_3 state, a tripod gait will be obtained. To prove this, all the possible initial conditions with the robot in a D_3 gait state should be considered. In a generic D_3 gait state like:

$$a > b < c > d < e > f < a \tag{3}$$

we can affirm (as explained in section 2) that legs 'b', 'd' and 'f' are in contact with the ground, but nothing can be said about the state of legs 'a', 'c' and 'e'. Four situations are possible attending to how many of these legs are not in contact with the ground.

To represent these four possibilities we need to enrich the gait state representation by including information about which legs are in contact with the ground and the time at which legs in the air will reach the ground. For instance, the gait state in (3) could be represented as:

$$x > x < t_1 > x < t_2 > x < x$$

where the 'x' represent legs in contact with the ground and t_i the time at which the corresponding leg will contact the ground with the convention that if $i < j$ then $t_i < t_j$. The evolution of these states is obtained using the following rules:

- $\ldots x > x < x \ldots \Longrightarrow \ldots x < t_n > x \ldots$
- $\ldots x < t_n > x \ldots \Longrightarrow \ldots x < x > x \ldots$

that represent the starting and the finishing of a protraction movement, respectively.

With this enriched gait state representation we can analyse the previously mentioned cases:

1. All legs are in contact with the ground. Then a tripod gait follows immediately.

2. One leg is in the air. The following sequence of situations occurs:

$$
\begin{array}{c}
x > x < t_1 > x < x > x < x \\
x > x < t_1 > x < x < t_2 > x \\
x > x < x > x < x < t_2 > x \\
x < t_3 > x < t_3 > x < t_2 > x \\
x < t_3 > x < t_3 > x < x > x \\
x < x > x < x > x < x > x
\end{array}
$$

   from which a tripod gait follows.

3. Two legs are in the air. In this case no leg can start a step until one of the legs that is in the air contacts with the ground and then the situation becomes the same as that of the previous case.

4. Three legs are in the air. In this case no leg can start a step until at least two of the legs in the air reach the ground, in which case we are in one of the previous situations.

So, we have shown that independently of the initial posture of the robot, if the gait state is of type D_3 and alternating steps are executed in constant time with no delays, then a tripod gait will be obtained.

This interesting property can be generalized to all states with R=3. We can prove that, if steps are executed in the conditions stated before, from any state with R=3, the tripod gait is achieved. Given the transitions between gait states with R=3 (figure 3), it is sure that any path through these states passes either by a state in D_3 (from which the tripod gait is obtained) or by a state in B_3. So, if with the same assumptions of the previous demonstration, it can be seen that from any initial conditions in B_3, a D_3 state is reached, then we can affirm the tripod gait is always achieved from any state with R=3. We have to analyse all possible initial conditions in B_3 and see if they derive into a D_3 gait state.

Consider a generic situation of kind B_3:

$$a > b < c > d > e < f < a$$

In this gait state legs 'b', 'd', 'e' and 'f' must be in contact with the ground and no information is available about the state of legs 'a' and 'c'. Two situations are possible:

1. All legs are in contact with the ground. Then:

$$
\begin{array}{c}
x > x < x > x > x < x < x \\
x < t_1 > x > x < t_1 > x < x \\
x < x > x > x < x > x < x \\
x < x > x < t_2 > x < t_2 > x
\end{array}
$$

2. A least on of the two legs is in the air. Then we have:

$$
\begin{array}{c}
- > x < - > x > x < x < x \\
- > x < - > x < t_3 > x < x
\end{array}
$$

In both cases a D_3 state is reached and from these, as we have seen before, the tripod gait is always obtained. Thus, from any gait state with R=3, if alternated steps are executed in constant time and with no unnecessary delays, the tripod gait will be the resulting gait independently of the initial posture of the robot.

5 Conclusions

It is well known that wave gaits are the most efficient and stable gaits that can be used to walk in straight line on a flat surface. Our method for gait analysis reveals that they all share a common property, i.e., they are executed with a constant value of R=3, and all steps performed are of the alternating type.

We have seen that performing only alternating steps, as far as its execution is not delayed, from any possible situation in which R=3 the tripod gait is reached after very few steps. The transition between different kinds of wave gaits can be also achieved by a very few number of alternating steps.

All this results point in the direction of favoring alternating steps which, intuitively, correspond to making steps with those legs that are in a more advanced phase of their power stroke, as is desirable.

However, we must note that there are many situations in which performing an alternating step turns out not to be the best thing to do. As an example, imagine that at a given point in time we want the robot to invert the advance direction to walk backwards. It is clear that the optimal strategy corresponds to a temporal inversion of the movements executed so far. This means that after the inversion, the leg that stepped last should be the first to step again, resulting in a repeated step. Other situations in which non-alternating steps may be convenient are:

- *Rough terrain:* If a foothold can not be found in the intended position, the leg must have to descend in a place much closer to its end of travel, with what its power stroke must finish before than expected. In extreme situations this may imply to step again before one (or both) of the neighboring legs.

- *Turning:* A similar problem is found when turning is achieved by shortening the length of the steps in the legs on a single side of the robot.

  Turning can also be achieved by keeping step lengths constant but setting different step frequences for the waves on each side. In this case, alternation of steps between legs on different sides is not possible.

In these cases, the execution of non-alternating steps can produce variations on the value of R. Once the conditions that impeded the execution of alternating steps end, the R value will remain constant, and potentially different from 3, in which case the further execution of only alternating steps would make impossible to reach the tripod or even any type of wave gait. To solve this situation some non-alternating steps with the only objective of recovering a state with R=3 must be issued.

Acknowledgments

This work has been partially supported by the *Comisión Interministerial de Ciencia y Tecnología* (CICYT) under the project "Navegación basada en visión de robots autónomos en entornos no estructurados" (TAP97-1209) and by a grant from the *Comissionat per a Universitats i Recerca de la Generalitat the Catalunya* to the first author.

References

[1] Beer R. D., Chiel H. J., Quinn R. D., Larsson P. (1992): "A Distributed Neural Network Architecture for Hexapod Robot Locomotion", *Neural Computation*, No. 4, pp. 356–365.

[2] Brooks R. A. (1989): "A Robot that Walks; Emergent Behaviors from a Carefully Evolved Network", *Neural Computation*, 1, pp. 253–262.

[3] Celaya E, Porta J. M. (1997): "A control Structure for the Locomotion of a Legged Robot on Difficult Terrain", *IEEE Robotics and Automation Magazine. Special Issue on Walking Robots*. To appear.

[4] Choi B. S., Song S. M. (1988): "Fully Automated Obstacle-Crossing Gaits for Walking Machines", *IEEE Transactions on System Man and Cybernetics*, Vol. 18, No. 6, pp. 952–964.

[5] Cruse H., Müller-Wilm U.,Dean J. (1994): "Artificial Neural Nets for Controlling a 6-Legged Walking System", Second International Conference on Simulation of Adaptive Behavior, *From Animals to Animats 2*, pp. 52–60.

[6] Ferrell C. (1993): "Robust Agent Control of an Autonomous Robot with Many Sensors and Actuators". MIT AI Lab. Technical Report 1443.

[7] McGhee R. B., Iswandhi G. I. (1979): "Adaptive Locomotion of a Multilegged Robot over Rough Terrain", *IEEE Transactions on System Man and Cybernetics*, Vol. SMC-9, No. 4, pp. 176–182.

[8] Ozguner F., Tsai S. J., McGhee R. B. (1984) "An Approach to the Use of Terrain-Preview Information in Rough-Terrain Locomotion by a Hexapod Walking Machine", *Int. Journal of Robotics Research*, Vol. 3, No. 2, pp. 134–146.

[9] Song S. M., Waldron K. J. (1987) "An Analytical Approach for Gait Study and its Application on Wave Gait", *Int. Journal of Robotics Research*, Vol. 6, No. 2, pp. 60-71.

[10] Wilson, D. M. (1966): "Insect Walking", Annual Rev. of Entomology, 11, pp. 103-122.