New compact representation of multiple-valued functions, relations, and non-deterministic state machines

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Talk outline

1. Current representations (cubes, BDDs, rough partitions)

2. Labeled Rough Partitions (lr-partitions)

3. Experimental results

4. Summary

Cube representation

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

y

x_1	x_2	x_3	x_4	y
0	0	0	0	0
0	0	0	1	1
0	0	1	1	0
0	0	1	0	1
0	1	0	0	1
0	1	0	1	0
0	1	1	1	1
0	1	1	0	0
1	1	0	0	0
1	1	0	1	1
1	1	1	1	0
1	1	1	0	1
1	0	0	0	1
1	0	0	1	0
1	0	1	1	1
1	0	1	0	0

2^n cubes

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	0	1	0	1
10	0	1	0	1

y

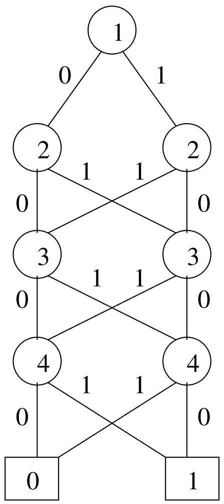
x_1	x_2	x_3	x_4	y
-	-	0	0	0
-	-	0	1	1
-	-	1	1	0
-	-	1	0	1

n cubes

BDD representation

x_1x_2 \ x_3x_4	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

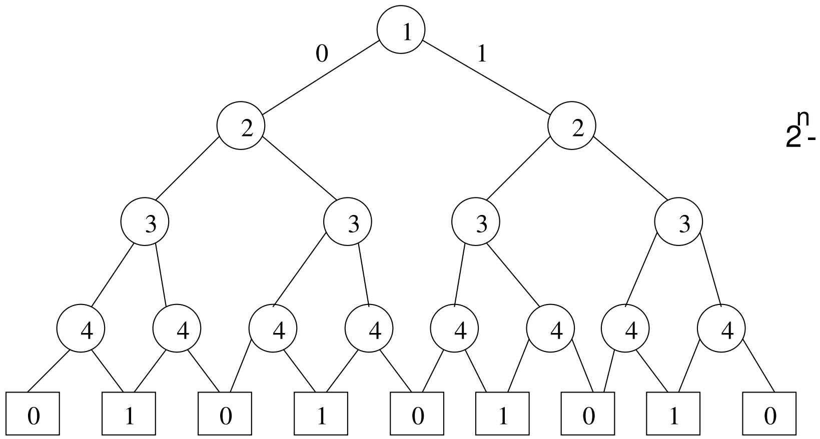
y



2n + 1 nodes

x_1x_2 \ x_3x_4	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	0	1	0	1
10	0	1	0	1

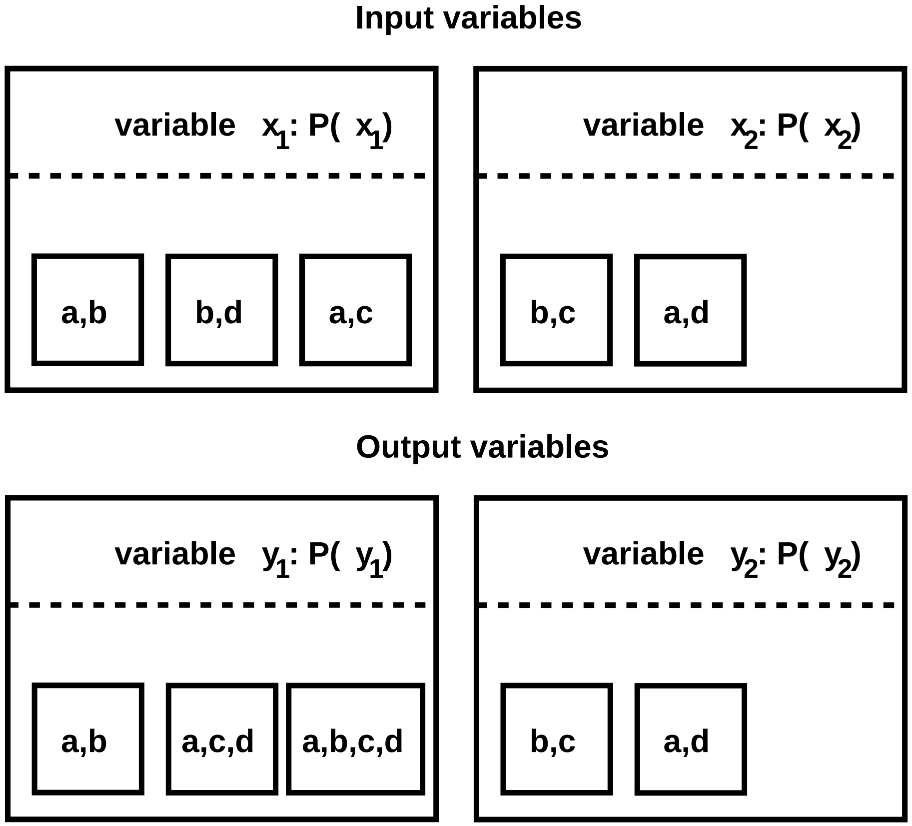
y



2^n - 1 nodes

Rough Partitions

	x_1	x_2	y_1	y_2
a	0,2	1	-	2
b	0,1	0	0,2	0
c	2	0	1,2	0
d	1	1	1,2	2



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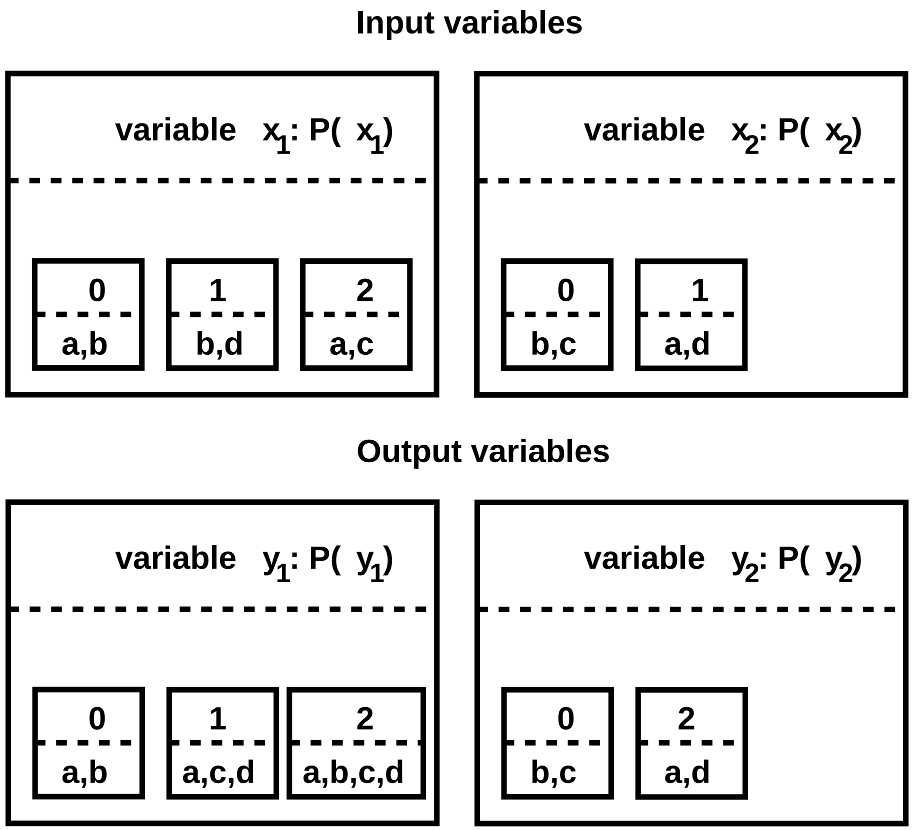
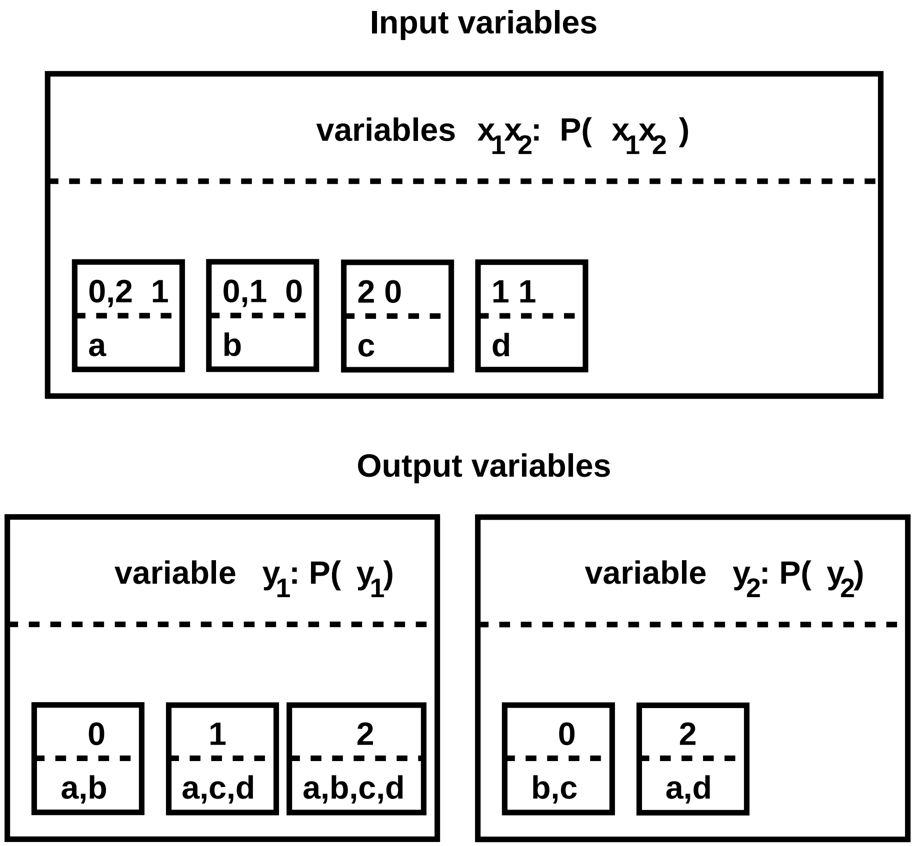
Lr-partitions representation 1

	x_1	x_2	y_1	y_2
a	0,2	1	-	2
b	0,1	0	0,2	0
c	2	0	1,2	0
d	1	1	1,2	2



Lr-partitions representation 2

	x_1	x_2	y_1	y_2
a	0,2	1	-	2
b	0,1	0	0,2	0
c	2	0	1,2	0
d	1	1	1,2	2



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Parity function

BDD representation: $2n+1$ nodes

	i/o	cubes	time [s]			#nodes		$\frac{\text{lr-BS}}{\text{BDD}}$	$\frac{\text{lr-BDD}}{\text{BDD}}$
			BDD	lr-BDD	lr-BS	lr-BDD	BDD		
p9	9/1	512	0.3	0.6	0.0	18	17	3.42	1.06
p10	10/1	1024	0.6	1.6	0.1	20	19	6.74	1.05
p11	11/1	2048	1.5	3.6	0.2	22	21	13.30	1.05
p12	12/1	4096	3.7	8.3	0.5	24	23	26.31	1.04
p14	14/1	16384	19.2	40.8	2.7	28	27	103.43	1.04
p16	16/1	65536	97.1	194.8	11.8	32	31	408.40	1.03

Multiply function

BDD representation: $O(2^{n/8}) = O(1.09^n)$ nodes

	i/o	#cubes	time [s]			#nodes		$\frac{\text{lr-BS}}{\text{BDD}}$	$\frac{\text{lr-BDD}}{\text{BDD}}$
			BDD	lr-BDD	lr-BS	lr-BDD	BDD		
m6	12/12	4096	11.0	22.4	1.0	1109	1103	1.01	1.0050
m7	14/14	16384	75.0	124.8	5.1	3116	3109	1.68	1.0020
m8	16/16	65536	461.2	663.7	23.2	8849	8841	2.70	1.0009
m9	18/18	262144	2419.5	3930.9	108.3	25063	25054	4.20	1.0004

Devadas function

BDD representation: $O(2^{n/2}) = O(1.41^n)$ nodes

Sum-of-products: n^2 product terms

	i/o	cubes	time [s]			#nodes		$\frac{\text{lr-BS}}{\text{BDD}}$	$\frac{\text{lr-BDD}}{\text{BDD}}$
			BDD	lr-BDD	lr-BS	lr-BDD	BDD		
d8	19/1	2038	23.3	12.8	0.3	1743	1610	0.29	1.08
d10	24/1	10308	251.2	104.8	2.7	3372	5331	0.55	0.63
d11	26/1	22631	831.6	249.9	7.0	2403	16445	0.42	0.15
d12	28/1	49151	2984.5	632.3	16.4	11930	20784	0.78	0.57

MCNC functions 1

	i/o	cubes	time [s]			#nodes		lr-BS / BDD	lr-BDD / BDD
			BDD	lr-BDD	lr-BS	lr-BDD	BDD		
apex1	45/45	1440	82.4	31.0	0.8	4877	1345	1.09	3.63
apex2	39/3	1576	42.2	31.0	0.6	5594	730	1.03	7.66
apex3	54/50	1036	-	20.1	0.7	3384	-	-	-
apex4	9/19	1907	1.9	13.8	0.3	4012	892	0.68	4.50
apex5	117/88	2710	7.2	120.9	4.6	7435	1130	5.59	6.58
seq	41/35	2014	-	47.5	1.0	6202	-	-	-
table3	14/14	1686	1.8	19.5	0.3	4311	778	0.69	5.54
table5	17/15	1600	1.7	21.8	0.3	4311	778	0.69	5.54
cps	24/109	855	3.2	10.3	0.5	2550	1072	1.21	2.38
cordic	23/2	2105	4.0	14.0	0.5	2136	61	9.84	35.02

MCNC functions 2

	i/o	cubes	time [s]			#nodes		$\frac{\text{lr-BS}}{\text{BDD}}$	$\frac{\text{lr-BDD}}{\text{BDD}}$
			BDD	lr-BDD	lr-BS	lr-BDD	BDD		
duke2	22/29	404	0.7	4.1	0.1	1364	392	0.60	3.48
e64	65/65	327	3.5	5.0	0.2	918	229	2.12	4.01
ex1010	10/10	1297	2.4	9.7	0.1	3605	1314	0.23	2.74
ex4p	128/28	654	2.5	19.8	0.8	2952	535	2.17	5.52
misex2	25/18	101	0.1	0.5	0.0	369	78	0.65	4.73
misex3	14/14	1391	4.0	15.5	0.2	4616	695	0.64	6.64
misex3c	14/14	1566	2.3	16.3	0.3	3976	499	1.00	7.97
pdc	16/40	822	1.6	7.9	0.2	2820	609	0.86	4.63
spla	16/46	837	1.3	8.5	0.2	2598	576	1.03	4.51
t481	16/1	841	0.7	4.5	0.1	1368	32	5.12	42.75
vg2	25/8	304	0.9	2.8	0.0	1139	301	0.38	3.78
alu4	14/8	1184	2.6	12.4	0.2	3478	800	0.37	4.35

MCNC functions 3

	i/o	cubes	time [s]			#nodes		lr-BS / BDD	lr-BDD / BDD
			BDD	lr-BDD	lr-BS	lr-BDD	BDD		
5xp1	7/10	141	0.0	0.4	0.0	388	55	0.51	7.05
9sym	9/1	158	0.1	0.6	0.0	485	26	0.70	18.65
bw	5/28	93	0.1	0.2	0.0	228	105	0.34	2.17
clip	9/5	271	0.2	1.3	0.0	759	92	0.47	8.25
ex5p	8/63	208	0.4	0.8	0.0	562	242	0.69	2.32
inc	7/9	94	0.0	0.3	0.0	265	68	0.26	3.90
rd53	5/3	67	0.0	0.1	0.0	180	19	0.34	9.47
rd73	7/3	274	0.1	1.1	0.0	661	35	0.91	18.89
rd84	8/4	511	0.2	2.3	0.0	928	48	1.45	19.33
sao2	10/4	137	0.1	0.5	0.0	459	89	0.26	5.16

Summary

- operations on lr-partitions can be entirely reduced to operations on sets of integer numbers
- lr-BDD are comparable in size to BDDs on parity functions
- lr-BDD are comparable in size to BDDs on multiply functions ($O(2^{n/8})$)
- lr-BDD are smaller in size than BDDs on Devadas functions ($O(2^{n/2})$)
- lr-BS are smaller in size than BDDs in 73% cases for MCNC benchmarks
- operations on lr-BS are much faster than on lr-BDD or BDD