

Finite State Machine Minimization

Advanced

*Methods based on triangular
table and binate covering*

Example 1. Minimize the following Mealy Finite State Machine

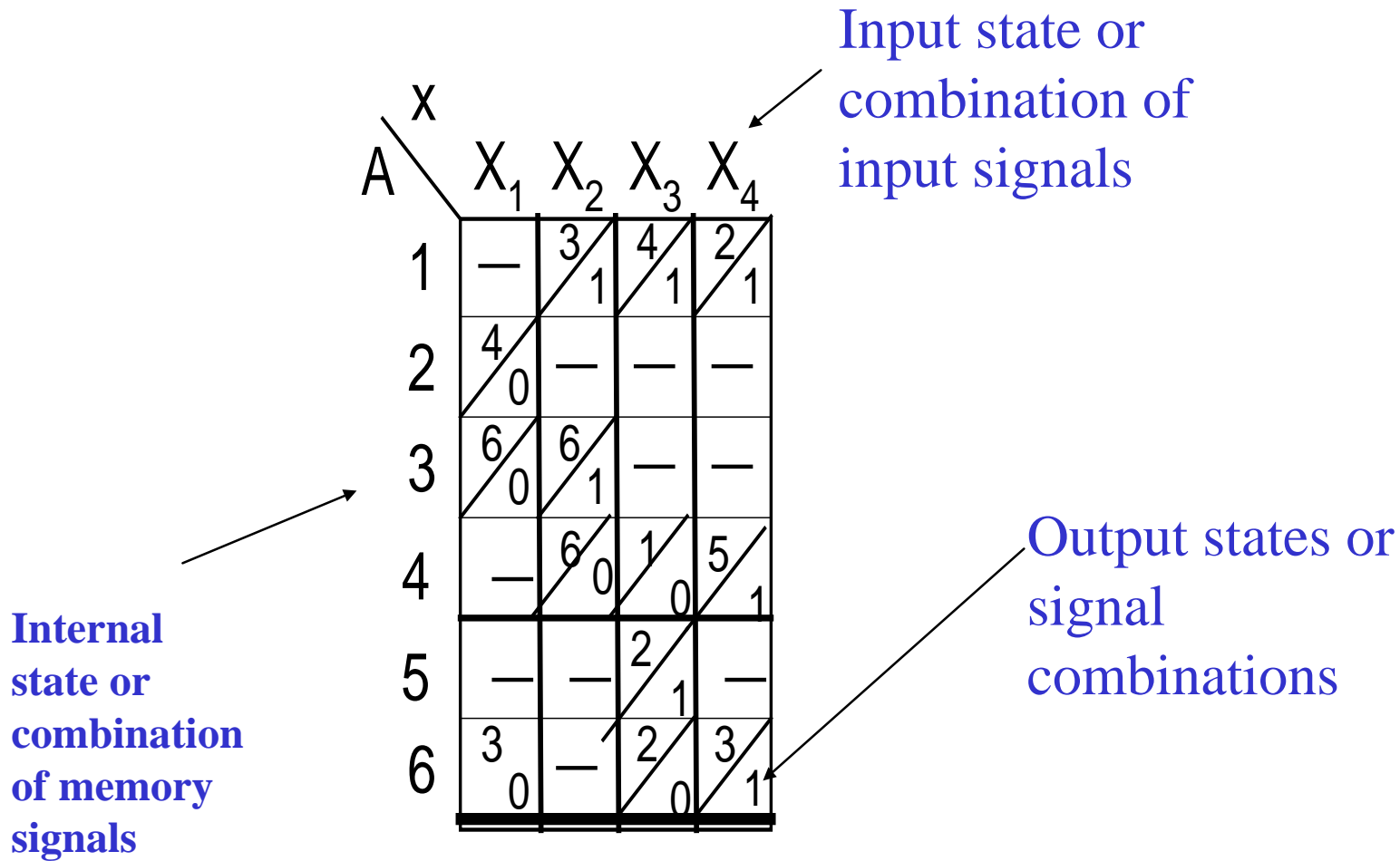


Fig 5.17bcd

A \ X		X			
		X ₁	X ₂	X ₃	X ₄
1	—	3 1	4 1	2 1	
2	4 0	—	—	—	
3	6 0	6 1	—	—	
4	—	6 0	1 0	5 1	
5	—	—	2 1	—	
6	3 0	—	2 0	3 1	

Compatibility Graph
obtained from the
Triangular Table

b)

		Triangular Table				
2						
3	3, 6	4, 6				
4	X		X			
5	2, 4			X		
6	X	3, 4		1, 2 3, 5	X	
	1	2	3	4	5	

c)

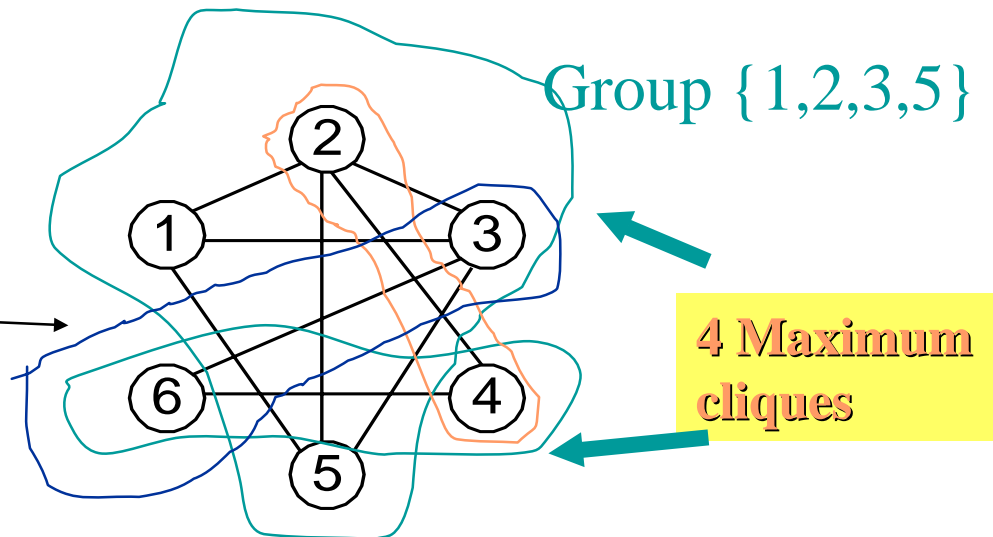
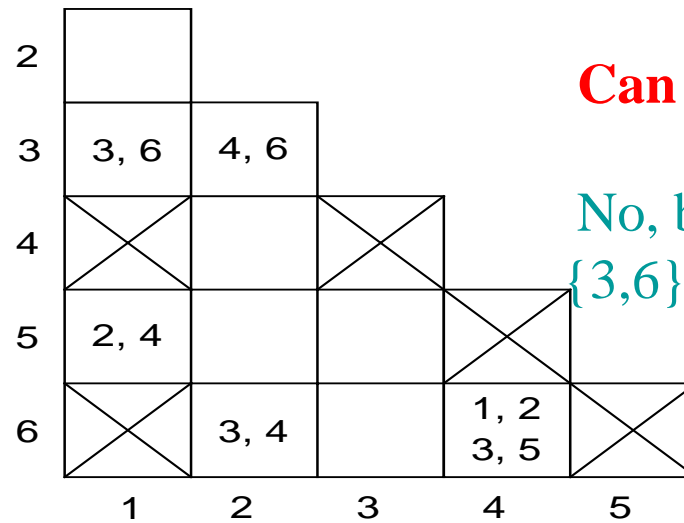


Fig 3.16

A \ X	X ₁	X ₂	X ₃	X ₄
1	-	3/1	4/1	2/1
2	4/0	-	-	-
3	6/0	6/1	-	-
4	-	6/0	1/0	5/1
5	-	-	2/1	-
6	3/0	-	2/0	3/1

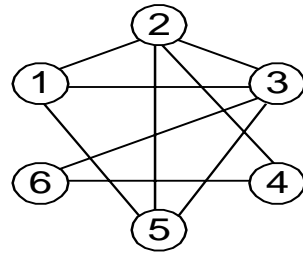
b)



Can I take {1,2,3,5} and {4,6}??

No, because {1,2,3,5} implies states {3,6} to be in one group.

c)



Solution is to split {1,2,3,5} to {1,2} and {3,5}

{1,2} implies nothing, {3,5} implies nothing, {4,6} implies {1,2} and {3,5}

I can take all max cliques but solution will be not minimal



Solution : {1,2}, {3,5}, {4,6}

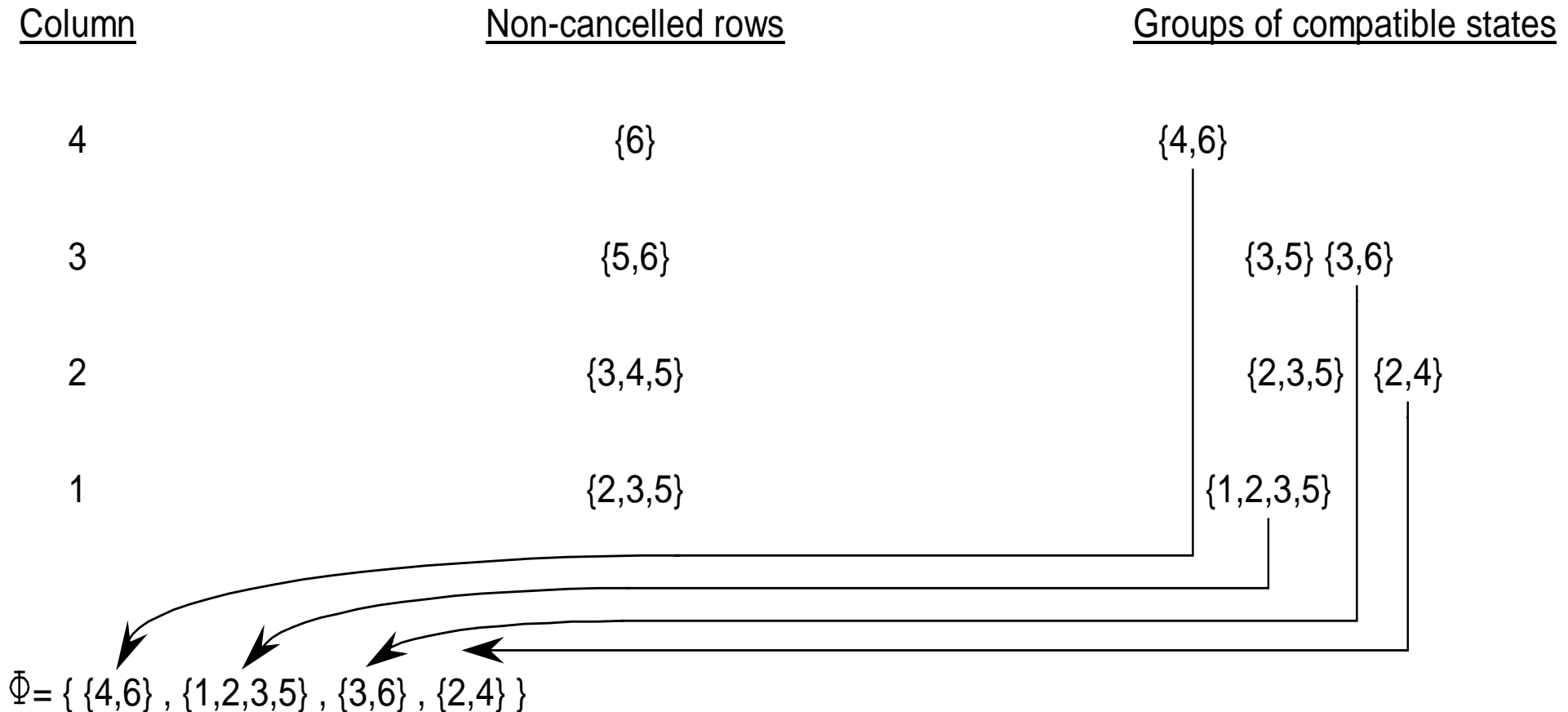
There are other solutions

But how I know to split this way? Heuristics!

In any case creating Maximum Compatible Groups is useful!

Systematic Method of Creating Maximum Compatible Groups

a)



This method is systematic and creates all maximum compatible groups (cliques)

For small FSMs you can find them by visual inspection

Fig5.17a

Complete and Closed Subgraph

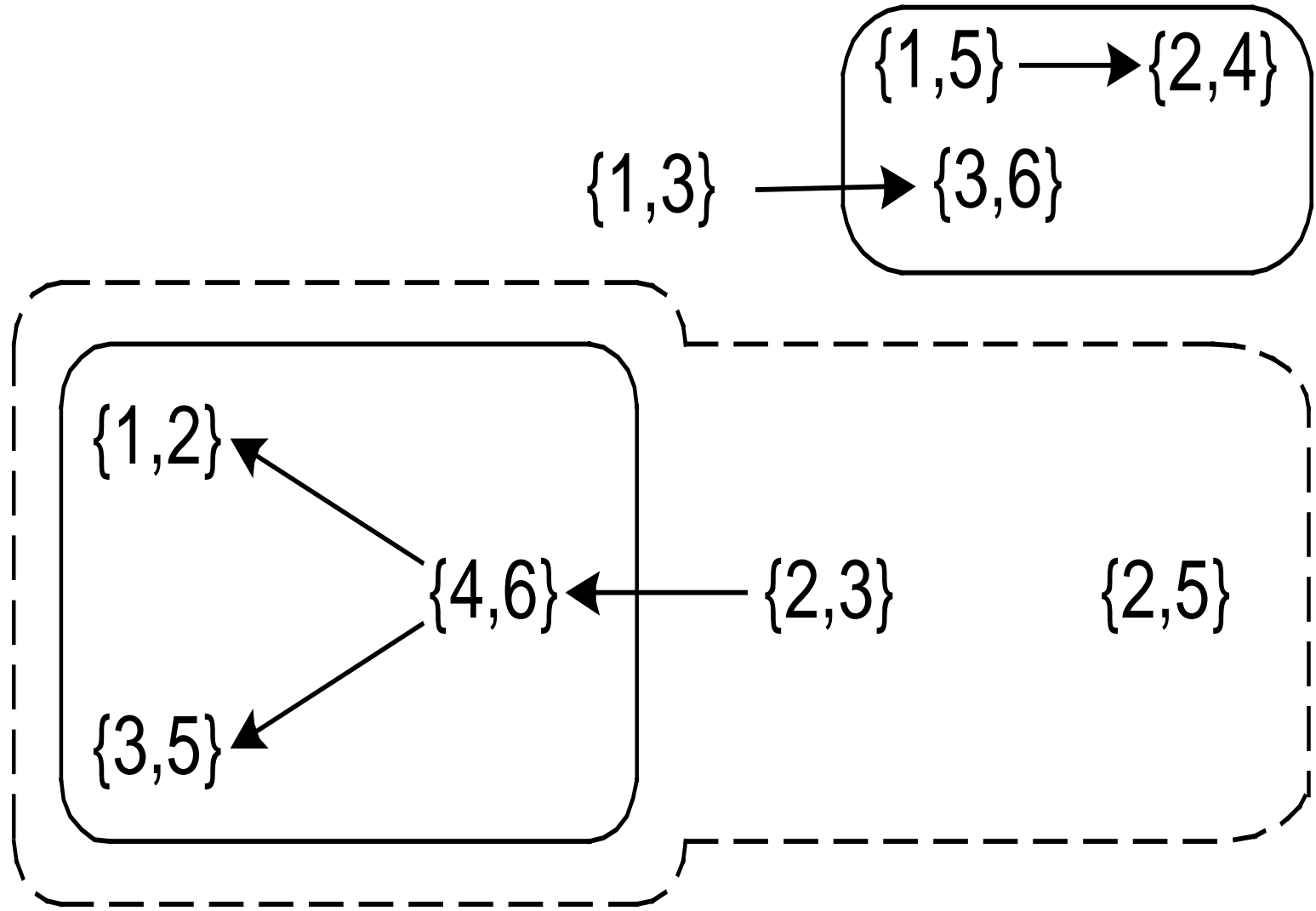
- **Complete** = all state numbers have been used at least one inside it
- **Closed** = there is no arrow going out of this graph

Closure graph for compatibility pairs

This method selects subsets of maximum cliques in order to satisfy the completeness and closure conditions for state numbers

This way we found other solutions.

Please draw machines



Combining groups of compatibles from the cover to single state

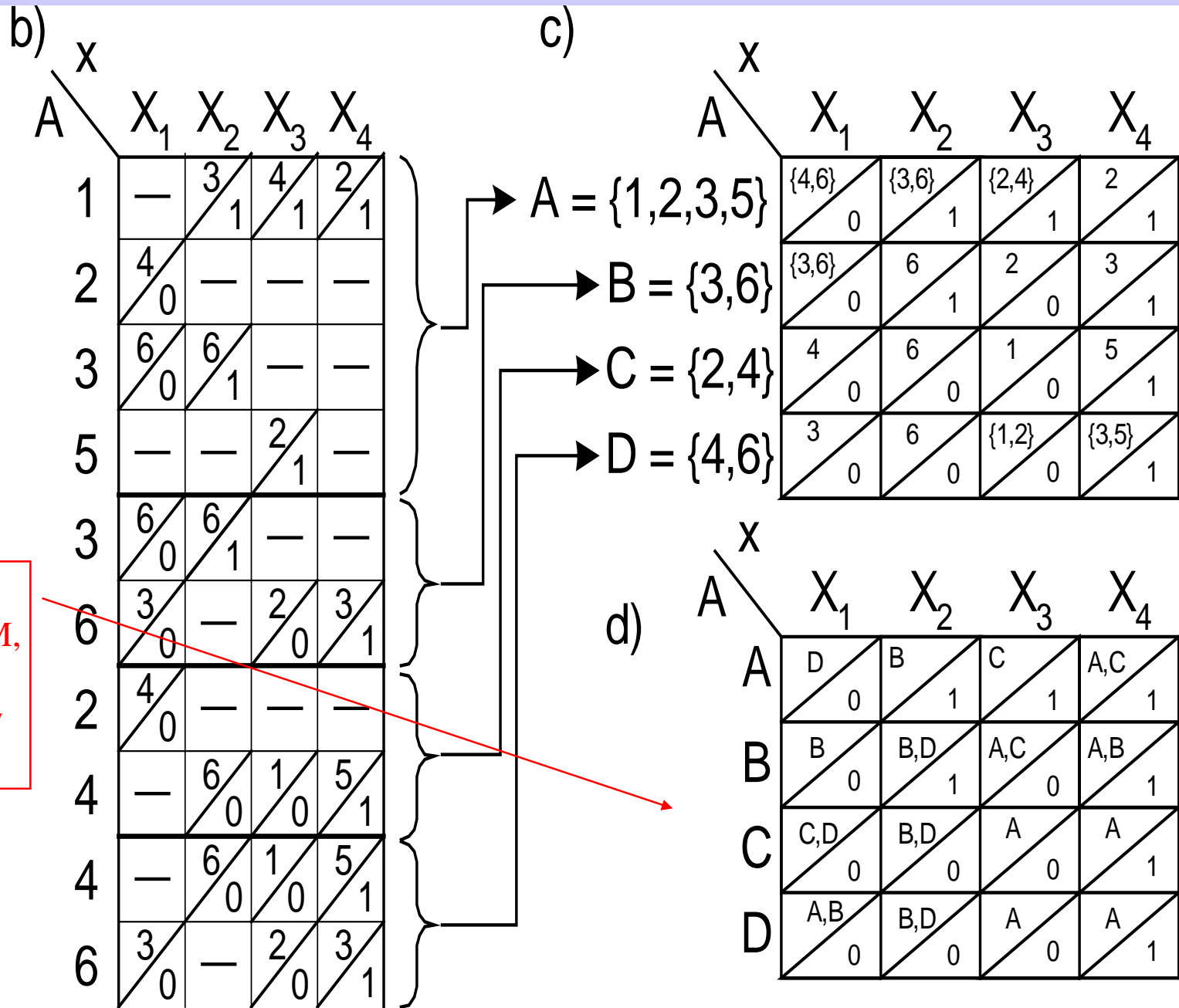
This is a final stage of state table minimization

It can be done with:

- 1) ALL groups of compatible states or
- 2) with the set of closed and closed groups of compatible states

Now let us go back to fast method, remember that it is not optimal

Combining ALL groups of compatibles from the cover to single state



Combining *ALL* groups of compatibles from the cover to single state

Select states from groups to create large groups of the same state

Select B
in whole
column

Select A
in whole
column

Select C

X	X ₁	X ₂	X ₃	X ₄
A	D 0	B 1	C 1	A,C 1
B	B 0	B,D 1	A,C 0	A,B 1
C	C,D 0	B,D 0	A 0	A 1
D	A,B 0	B,D 0	A 0	A 1

X	X ₁	X ₂	X ₃	X ₄
A	D 0	B 1	C 1	A,C 1
B	B 0	E,D 1	A,C 0	A,B 1
C	C,D 0	E,D 0	A 0	A 1
D	A,B 0	E,D 0	A 0	A 1

As you see, it is a good idea to combine FSM minimization and state assignment. Many methods are based on this idea.

Creating new table by combining states from groups of compatible states

- The same method of combining states can be applied to any set of compatible and closed

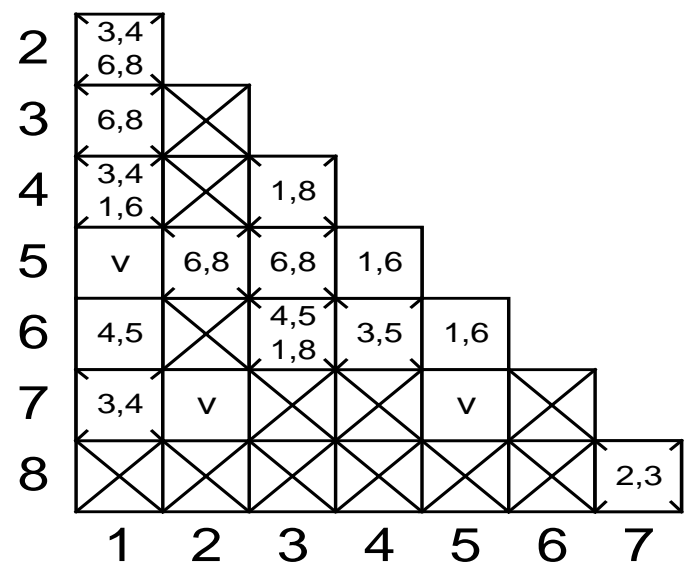
Problem: Find FSM table for which the following triangular table exists:

2					
3	X	X			
4	X	X	1,2		
5	3,4 2,6	3,4 2,6	X	X	
6	3,4 2,5	3,4 2,5	X	X	
	1	2	3	4	5

Fig.5.18

Example 3 of FSM Minimization

b)

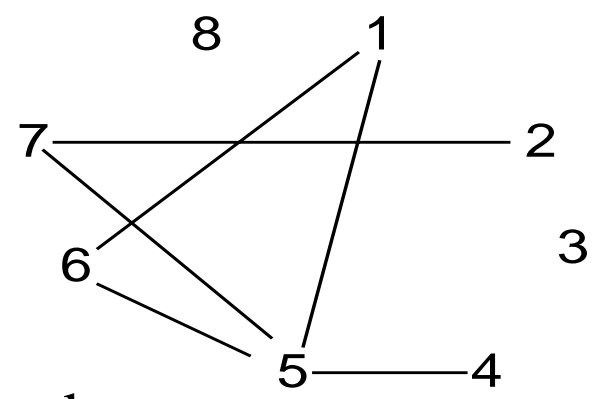


1	4/-	6/0
2	3/1	8/0
3	4/0	8/0
4	3/0	1/0
5	-/-	6/0
6	5/0	1/0
7	3/1	-/-
8	2/1	1/1

- {1,6}
- {2,7}
- {3}
- {4,5}
- {8}

4/0	1/0
3/1	8/0
4/0	8/0
3/0	1/0
2/1	1/1

c)



d)

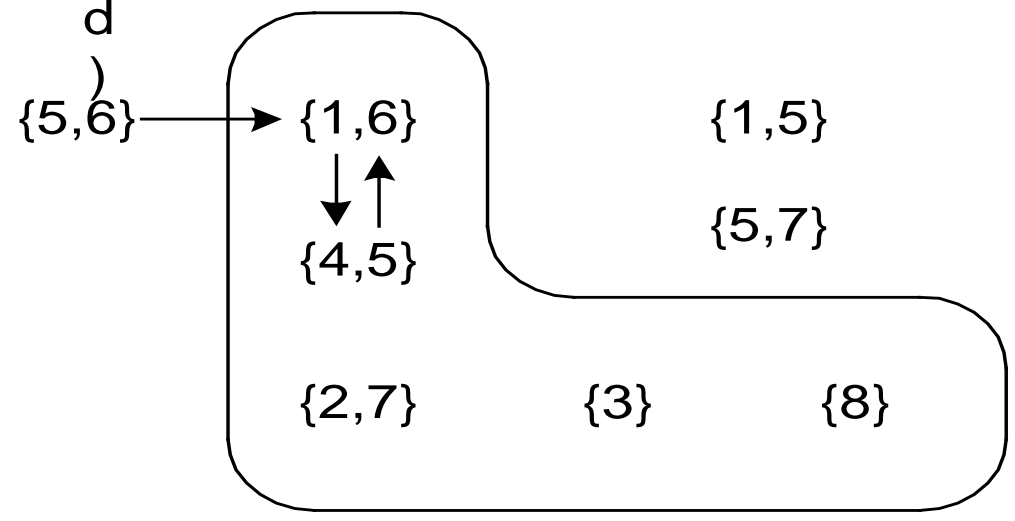


Fig.5.21.bcd