Fast Fourier Transform

S. Schüppel

The Fast Fourier Transform

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Introduction

Use of FFT algorithm

- originally discovered by Gauß in 1804
- 'rediscovery' by Danielson and Lanczos in 1942
- generally know through Cooley / Tukey in mid 60s
- DFT: $O(n^2)$ vs. FFT: $O(n\ log_2(n))$

$$X = \sum_{k=0}^{N-1} x_k\ e^{-i\left(\frac{2\pi}{N}\right)kn} \qquad (1)$$

- use $N = 2^p$ number of samples
- divide and conquer algorithm

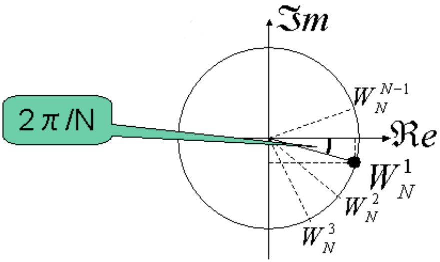
Introduction

Twiddle Factor

$$e^{-i\frac{2\pi}{N}} = W_N \tag{2}$$

$$X(n) = \sum_{k=0}^{N-1} x_k \, e^{-i\left(\frac{2\pi}{N}\right)kn} = \sum_{k=0}^{N-1} x_k \, W_N^{kn} \tag{3}$$

- Euler's Law: $e^{i\theta} = \sin(\theta) + i\cos(\theta) = \sin(\frac{2\pi}{N}) - i\cos(\frac{2\pi}{N})$



Fast Fourier Transform

Divide and Conquer

$$X(n) = \sum_{k=0}^{(N/2)-1} x_{2k}\ e^{-i\left(\frac{2\pi}{N}\right)(2k)n} + \sum_{k=0}^{(N/2)-1} x_{2k+1}\ e^{-i\left(\frac{2\pi}{N}\right)(2k+1)n}$$

$$= \sum_{k=0}^{(N/2)-1} x_{2k}\ e^{-i\left(\frac{2\pi}{N}\right)(2k)n} + e^{-i\left(\frac{2\pi}{N}\right)n} \sum_{k=0}^{(N/2)-1} x_{2k+1}\ e^{-i\left(\frac{2\pi}{N}\right)(2k)n}$$

$$= \sum_{k=0}^{M} x_{2k}\ W_M^{kn} + W_N^n \sum_{k=0}^{M} x_{2k+1}\ W_M^{kn} \qquad \left(M = \frac{N}{2}\right) \tag{4}$$

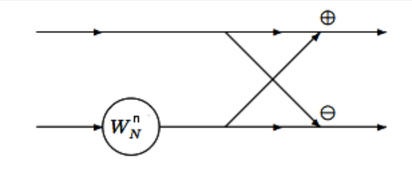
Solution through recursion

$$N = 1 \qquad \Longrightarrow \qquad X(0) = x_0 W_1^0 = x_0$$

Basic Building Block - Butterfly

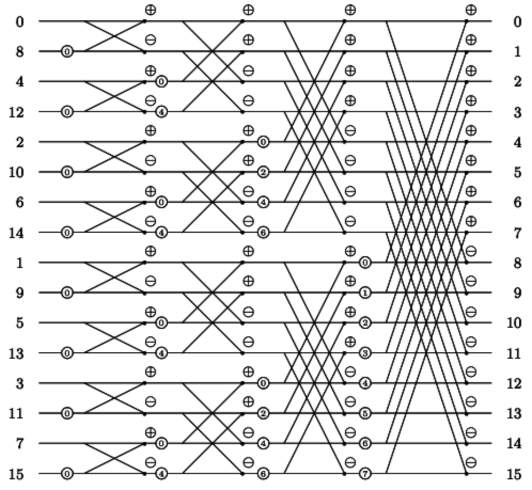
$$X(n) = X_{2k} + W_N^n X_{2k+1} \quad \left(M = \frac{N}{2}\right) \tag{5}$$

$$X(n+M) = X_{2k} - W_N^n X_{2k+1} \tag{6}$$



Fast Fourier Transform - N = 16

bit-reversed order !



FFT IP-Cores

Why using IP-Cores?

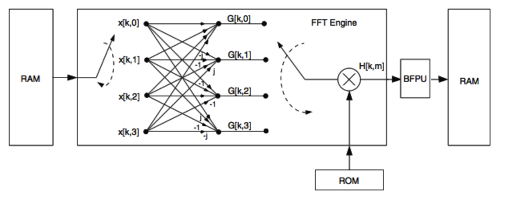
- Easy to Implement
- Free (Altera)
- Optimized in Speed and Area

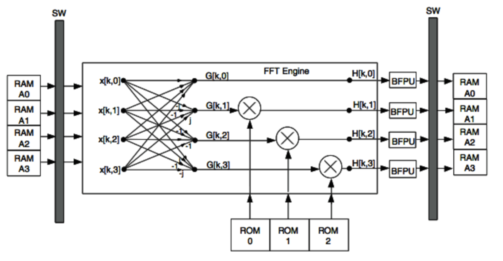
Altera Features

- Matlab Model
- radix-4 and mixed radix-4/2
- different Input and Output orders
- transformation length $2^m \quad 6 \leqslant m \leqslant 14$

Inside of the Altera Core

Single and Quad radix-4 Output





Recommended Literature

- "Fourier Transform for Pedestrians" published by Springer Verlag