# Project 3

## Pareto optimality GA and QC synthesis

**GOAL:** Using Pareto-optimality in GA in order to explore the results, constraints and possibilities given by this approach

In this project you have to make a search for quantum gates using a Pareto optimal GA. Pareto optimality is used for multiobjective optimization and you have discussed it in the class. Here I will give you only some hints and goals of this project. As you have seen in Mr. Lukac presentation on Thursday we are evolving quantum circuits and we are using a composed fitness function:

$$F = \alpha(1 - \frac{Error}{Max\_error}) + \beta\frac{1}{Cost}$$

with F is the resulting fitness, the first left-hand term is the error evaluation, the second one is the cost of the circuit and α and β are scaling factors so as one can decide to either minimize more for the cost of the circuit or for the correctness. In this project you have to consider as many parameters as you can imagine to be useful for the evolution of quantum circuits. The evaluation of individuals will be then done via the comparison of each individual with all others and the result will be the Goldberg ranking. This can be described as:

- Each individual/circuit will have n parameters $X = x_0 \ldots x_{n-1}$
- For each generation compare all parameters of one individual to all others in the population and assign point according to this rule
    - If $X_i \geq X_j$ then add +1 to the individual i
    - Else add nothing.
- Select individuals using a selection method of your choice to select from a Ranked population

You will be given a set of gates to evolve so as a set of gates to use. The software will be ready and your task is only to select parameters to be used. The minimal requirement is to use two parameters Error and Cost. However you can invent new parameters such as complexity of the circuit, length, and so on.

As you will see and perhaps discover you will have to scale the influence and the weight of certain parameters. This means that we are looking mainly for a correct circuit and beside we want to minimize its cost, complexity, etc. Possible ways to do this are:

- scaling coefficients: comparing certain parameters multiplied by a coefficient < 0 result in an increase of the fitness
- constant multiplication: similar effect as with coefficients

The following example will explain the above presented idea of how to parameterize GA using pareto optimality. Assume we want to compare two individuals $I_i$ and $I_j$ each with a set of parameters $X = (x_0,x_1,x_3,x_4)$. Also assume the most important parameter is $x_0$ and $x_1$. For this we will compare the first two parameters unmodified and the remaining two will be biased by a coefficient of 0.8 so as then we will be comparing $x_{2i}>0.8x_{2j}$. This has for consequence that an element that is not better in all parameters will earn a +1 if it satisfied the previous condition. Another example is to share a win between more than one parameter. Again let's take our previous individuals $I_i$ and $I_j$ with their respective sets of parameters. This time we select another approach. For each parameter $x_{ni}$ that is greater than $x_{nj}$ we assign a parameter being a numerical value in [0,1]. And the increasing of the rank is now based on a threshold. Assume we assign 1,1,0.7 and 0.5 to $x_0$,$x_1$,$x_2$, and $x_4$ respectively and we set a threshold to 2. Now assuming that $x_{0i}$ and $x_{1i}$ are greater than $x_{0j}$ and $x_{1j}$ we can add +1 to the rank, but for the case $x_{0i}$ and $x_{2i}$ are greater than $x_{0j}$ and $x_{2j}$ we keep the individual rank not increased. Such a use of parameters reflects the importance given to different coefficients and consequently can be well used to evolve complex structures such as quantum circuits.

**Expected results:**

- Detailed description of settings you have been using
- Report on results, problems, deceptions and achievements
- Analysis of the Pareto optimality for the quantum circuit synthesis
- Graphical representation of results