COS 495 - Lecture 17
Autonomous Robot Navigation

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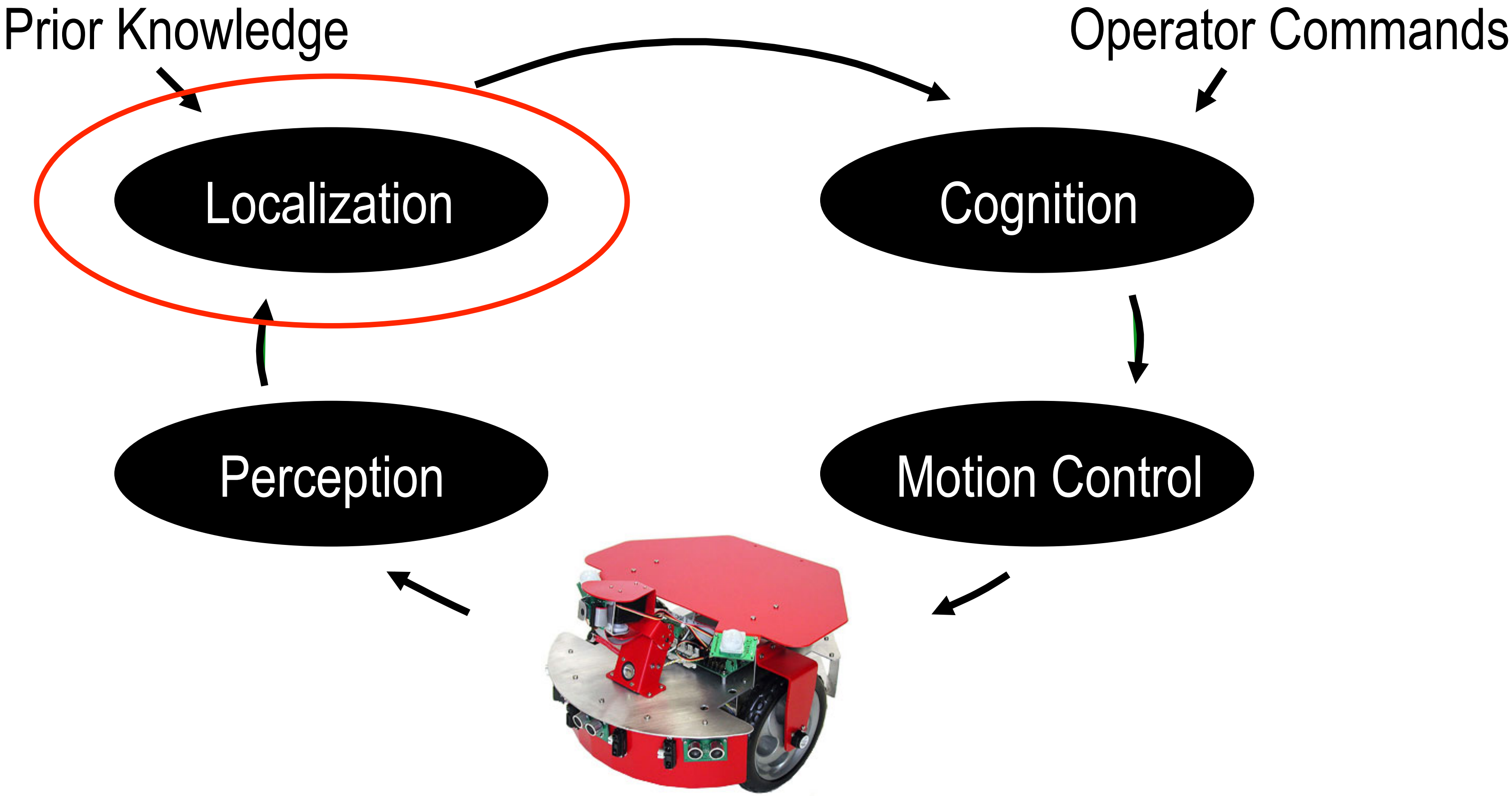
Figures courtesy of Siegwart & Nourbakhsh

Control Structure



Extended Kalman Filter Localization

1. EKF Localization Overview
2. EKF Prediction
3. EKF Correction
4. Algorithm Summary

Extended Kalman Filter Localization

- Robot State Representation
  - State vector to be estimated, $\boldsymbol{x}$

    e.g.

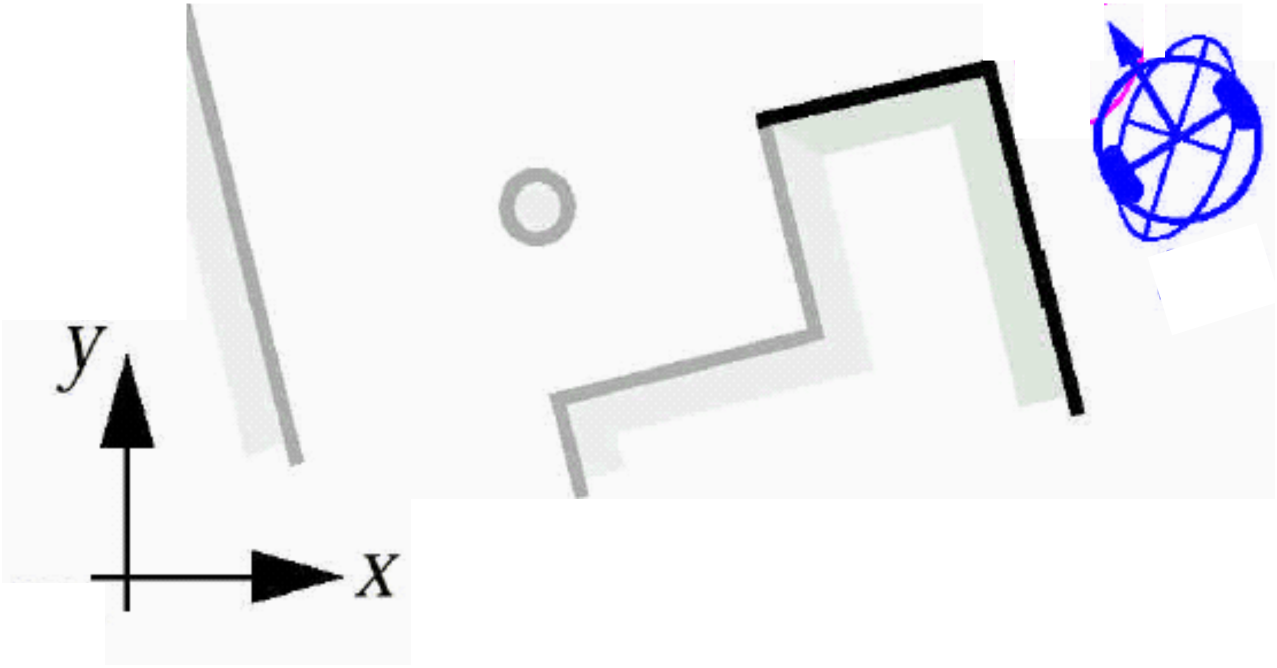
$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

  - Associated Covariance, $\boldsymbol{P}$

$$\boldsymbol{P} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} \end{bmatrix}$$

Extended Kalman Filter Localization

1. Robot State Representation



Extended Kalman Filter Localization

- Iterative algorithm
  1. Prediction – Use a motion model and odometry to predict the state of the robot and its covariance

     $$x'_t \quad P'_t$$

  2. Correction - Use a sensor model and measurement to predict the state of the robot and its covariance

     $$x_t \quad P_t$$

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EKFL Prediction Step

- Motion Model
  - Lets use a general form of a motion model as a discrete time equation that predicts the current state of the robot given the previous state x_{t-1} and the odemetry u_t

$$x'_t = f(x_{t-1}, u_t)$$

EKFL Prediction Step

- Motion model
  - For our differential drive robot…

$$\boldsymbol{x_{t-1}} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}$$

$$\boldsymbol{u_t} = \begin{bmatrix} \Delta s_{r,t} \\ \Delta s_{l,t} \end{bmatrix}$$

EKFL Prediction Step

- Motion model
  - And the model we derived…

$$x'_t = f(x_{t-1}, u_t) = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \Delta s_t \cos(\theta_{t-1} + \Delta\theta_t/2) \\ \Delta s_t \sin(\theta_{t-1} + \Delta\theta_t/2) \\ \Delta\theta_t \end{bmatrix}$$

$$\Delta s_t = (\Delta s_{r,t} + \Delta s_{l,t})/2$$

$$\Delta\theta_t = (\Delta s_{r,t} - \Delta s_{l,t})/b$$

EKFL Prediction Step

- Covariance
  - Recall, the propagation of error equation…

$$\left.\begin{array}{r} X \sim N(\mu,\Sigma) \\ Y = AX + B \end{array}\right\} \Rightarrow \quad Y \sim N(A\mu + B, A\Sigma A^T)$$

EKFL Prediction Step

- Covariance
  - Our equation *f()* is not linear, so to use the property we will linearize with first order approximation

$$x'_t = f(x_{t-1}, u_t)$$
$$\approx F_{x,t} x_{t-1} + F_{u,t} u_t$$

where

$F_{x,t}$ = *Derivative of f with respect to state* x_{t-1}

$F_{u,t}$ = *Derivative of f with respect to control*

EKFL Prediction Step

- Covariance
  - Here, we linearize the motion model f to obtain

$$P'_t = F_{x,t} P_{t-1} F_{x,t}^T + F_{u,t} Q_t F_{u,t}^T$$

where

Q_t = *Motion Error Covariance Matrix*

$F_{x,t}$ = *Derivative of f with respect to state* $\mathbf{x}_{t-1}$

$F_{u,t}$ = *Derivative of f with respect to control* $\mathbf{u}_t$

EKFL Prediction Step

- Covariance

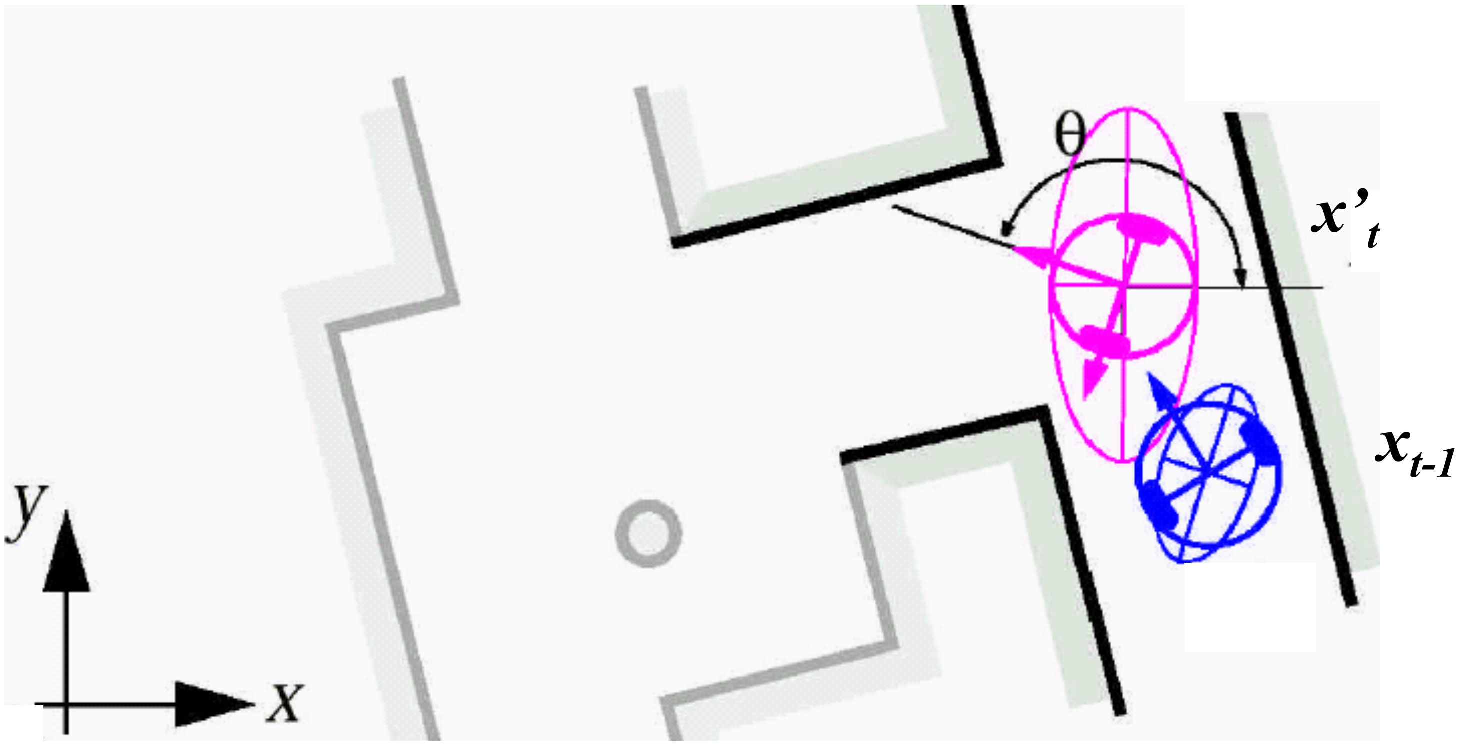
$$Q_t = \begin{bmatrix} k\,|\Delta s_{r,t}| & 0 \\ 0 & k\,|\Delta s_{l,t}| \end{bmatrix}$$

$$F_{x,t} = \begin{bmatrix} df/dx_t & df/dy_t & df/d\theta_t \end{bmatrix}$$

$$F_{u,t} = \begin{bmatrix} df/d\Delta s_{r,t} & df/d\Delta s_{l,t} \end{bmatrix}$$

EKFL Prediction Step

1. Motion Model



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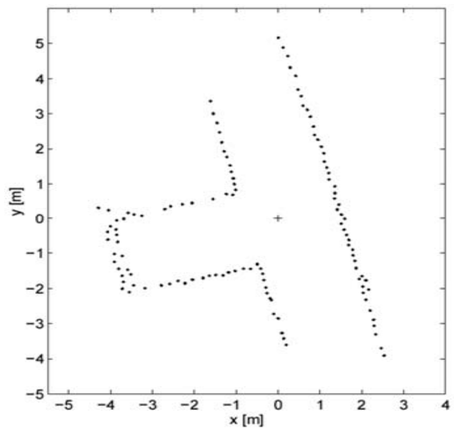
EKFL Correction Step

- Innovation
  - We **correct** by comparing current measurements z_t with what we expect to observe $z_{exp,t}$ given our predicted location in the map M.
  - The amount we correct our state is proportional to the **innovation** v_t

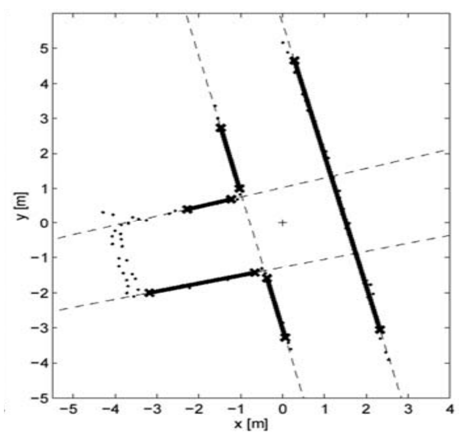
$$v_t = z_t - z_{exp,t}$$

EKFL Correction Step

- The Measurement
  - Assume our robot measures the relative location of a wall i extracted as line



$$\boldsymbol{z}^i_t = \begin{bmatrix} \alpha^i_t \\ r^i_t \end{bmatrix} \qquad \boldsymbol{R}^i_t = \begin{bmatrix} \sigma^i_{\alpha\alpha,t} & \sigma^i_{\alpha r,t} \\ \sigma^i_{r\alpha,t} & \sigma^i_{rr,t} \end{bmatrix}$$



EKFL Correction Step

- The Measurement
  - Assume our robot measures the relative location of a wall i extracted as line

$$z^i_t = \begin{bmatrix} \alpha^i_t \\ r^i_t \end{bmatrix} = g(\rho_1, \rho_2, \ldots, \rho_n, \beta_1, \beta_2, \ldots, \beta_n)$$

$$\alpha = \frac{1}{2}\operatorname{atan}\left(\frac{\sum w_i \rho_i^2 \sin 2\beta_i - \frac{2}{\sum w_i} \sum\sum w_i w_j \rho_i \rho_j \cos\beta_i \sin\beta_j}{\sum w_i \rho_i^2 \cos 2\beta_i - \frac{1}{\sum w_i} \sum\sum w_i w_j \rho_i \rho_j \cos(\beta_i + \beta_j)} \right)$$

$$r = \frac{\sum w_i \rho_i \cos(\beta_i - \alpha)}{\sum w_i}$$

EKFL Correction Step

- The Measurement

$$R^i_t = \begin{bmatrix} \sigma^i_{\alpha\alpha,t} & \sigma^i_{\alpha r,t} \\ \sigma^i_{r\alpha,t} & \sigma^i_{rr,t} \end{bmatrix}$$

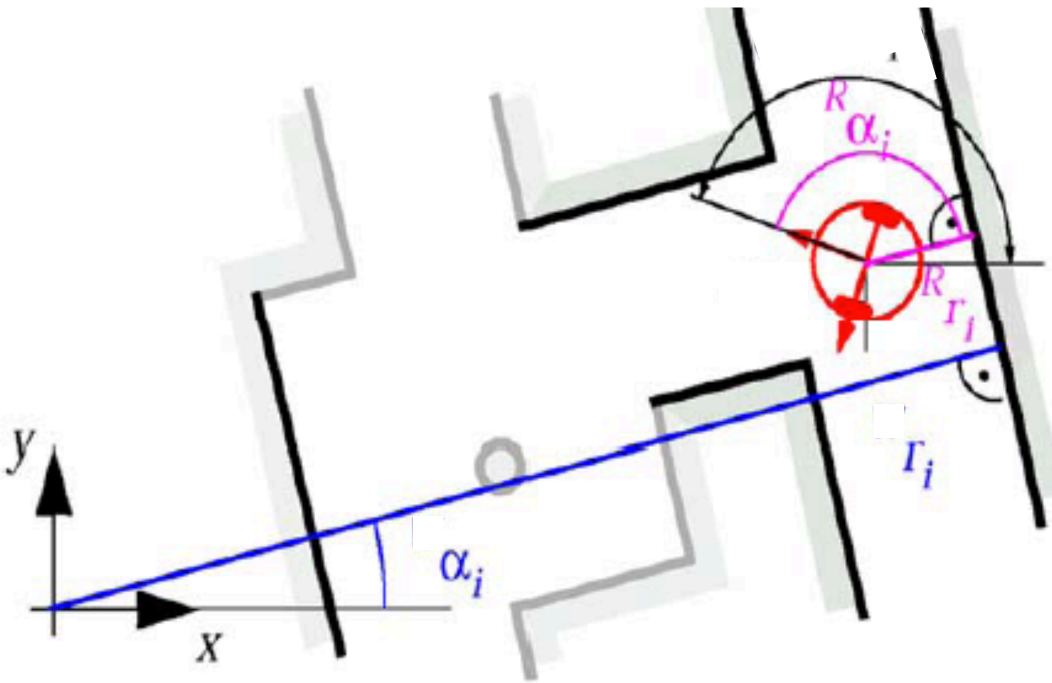
$$= G_{\rho\beta,t}\, \Sigma_{z,t}\, G_{\rho\beta,t}^{\;T}$$

where

$\Sigma_{z,t}$ = *Sensor Error Covariance Matrix*

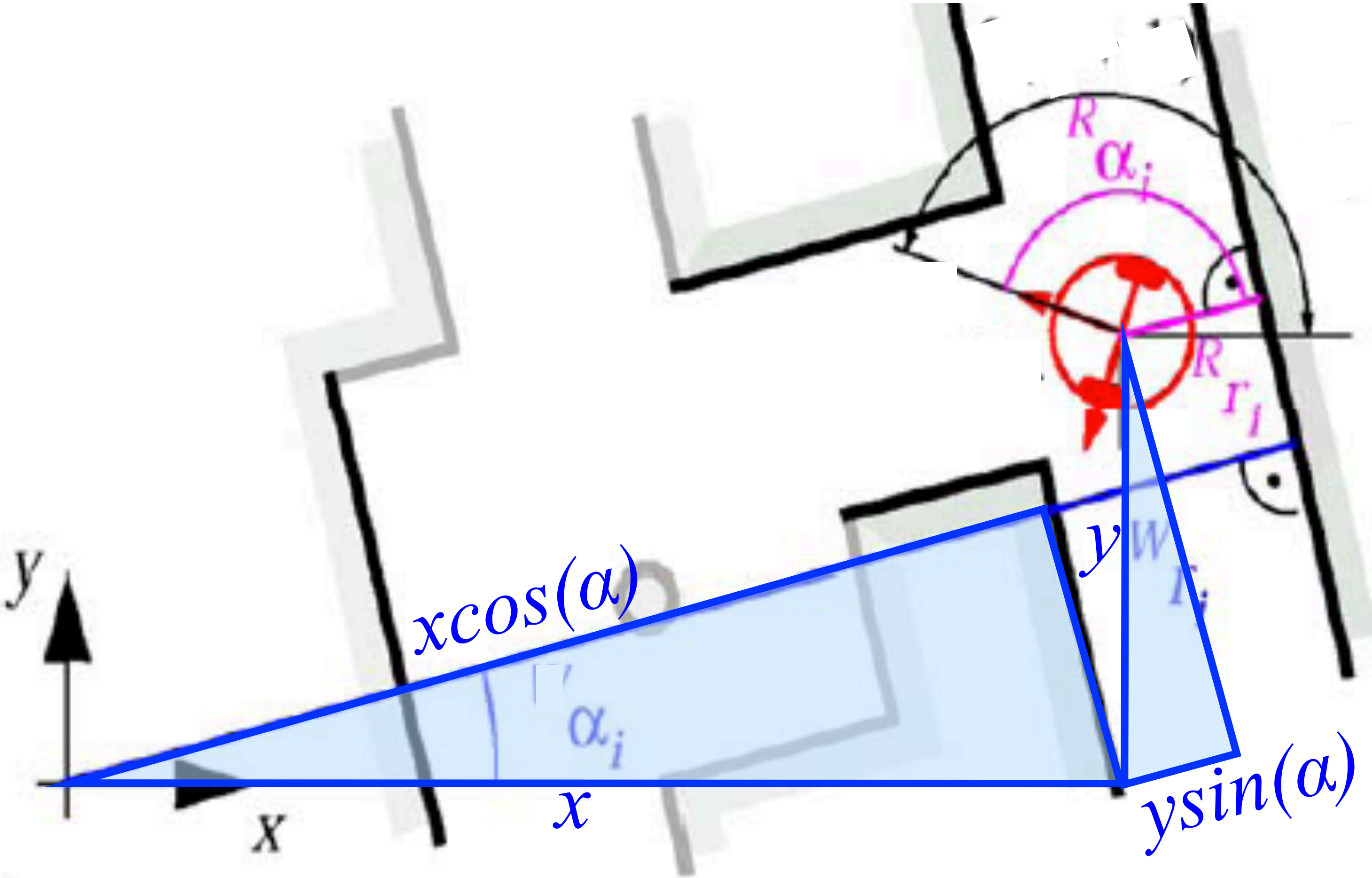
$G_{\rho\beta,t}$ = *Derivative of g() wrt measurements* ρ_t, β_t

EKFL Correction Step



$$\mathbf{z}^{i}_{exp,t} = h^{i}(\mathbf{x'}_{t}, \mathbf{M}) = \begin{bmatrix} \alpha^{i}_{M} - \theta'_{t} \\ r^{i} - x'_{t}\cos(\alpha^{i}_{M}) - y'_{t}\sin(\alpha^{i}_{M}) \end{bmatrix}$$

EKFL Correction Step



EKFL Correction Step

- The covariance associate with the innovation is

$$\Sigma_{IN,t} = H^{i}_{x,t} P'_{t} H^{i\,T}_{x,t} + R^{i}_{t}$$

where

R^{i}_{t} = *Line Measurement Error Covariance Matrix*

$H^{i}_{x,t}$ = *Derivative of h with respect to state* $\mathbf{x}_t$

EKFL Correction Step

- Final updates
  - Update the state estimate

$$x_t = x'_t + K_t\, v_t$$

  - Update the associated covariance matrix

$$P_t = P'_t - K_t\, \Sigma_{IN,t} K_t^T$$

  - Both use the Kalman gain Matrix

$$K_t = P'_t\, H_{x',t}^T\, (\Sigma_{IN,t})^{-1}$$

EKFL Correction Step

- Compare with single var. KF
  - Update the state estimate

$$\hat{x}_t = \hat{x}_{t-1} + K_t (z_t - \hat{x}_{t-1})$$

  - Update the associated covariance matrix

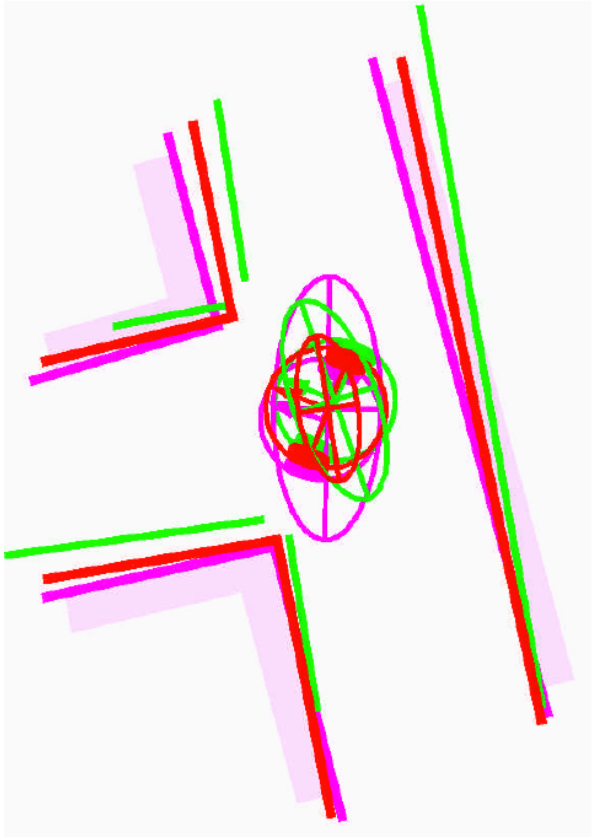
$$\sigma_t^2 = \sigma_{t-1}^2 - K_t \sigma_{t-1}^2$$

Both use the Kalman gain Matrix

$$K_t = \frac{\sigma_{t-1}^2}{\sigma_{t-1}^2 + \sigma_t^2}$$

EKFL Correction Step

- Final updates
  - By fusing the prediction of robot position (magenta) with the innovation gained by the measurements (green) we get the updated estimate of the robot position (red)



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EKFL Summary

Prediction

1. $x'_t = f(x_{t-1}, u_t)$
2. $P'_t = F_{x,t} P_{t-1} F_{x,t}^T + F_{u,t} Q_t F_{u,t}^T$

Correction

3. $z^i_{exp,t} = h^i(x'_t, M)$
4. $v_t = z_t - z_{exp,t}$
5. $\Sigma_{IN,t} = H^i_{x',t} P'_t H^i_{x',t}{}^T + R^i_t$
6. $x_t = x'_t + K_t v_t$
7. $P_t = P'_t - K_t \Sigma_{IN,t} K_t^T$
8. $K_t = P'_t H_{x',t}^T (\Sigma_{IN,t})^{-1}$