

## Mobile Robot Position Estimation Using the Kalman Filter

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### Abstract

*The Kalman filters have been widely used for mobile robot navigation and system integration. So that it may operate autonomously, a mobile robot must know where it is. Accurate localization is a key prerequisite for successful navigation in large-scale environments, particularly when global models are used, such as maps, drawings, topological descriptions, and CAD models. The objective of this paper is to implement the Kalman filter (KF) and the extended Kalman Filter (EKF) for determining the position of a mobile robot. Based on the results of the study, from the figures can be seen that despite of the errors present in measurements, the filters can perform quite well in estimating, the robot's true position.*

### 1. Introduction

Filtering is a very used method in engineering and embedded systems. A good filtering algorithm can reduce the noise from signals while retaining the useful information. The Kalman filter (KF) [12] is a mathematical tool that can estimate the variables of a wide range of process. It estimates the states of a linear system. This type of filter works very well in practice and that is why it is often implemented in embedded control system and because we need an accurate estimate of the process variables. The KF has been widely used for mobile robot navigation [3, 5, 6] and system integration. So that it may operate autonomously, a mobile robot must know where it is. Accurate localization is a key prerequisite for successful navigation in large-scale environments, particularly where global models are used. The KF has many limitations and that is why many authors proposed various fixes and modifications to better estimate the process variables [7].

The extended Kalman filter (EKF) is a variation of the standard KF to nonlinear systems. It works by linearizing the nonlinear state dynamics and measurement models. Like the KF, the EKF has been successfully applied in the field of position estimation for mobile robots [2, 5, 8, 9, 11]. One of the most important applications of the EKF is an simultaneous localization and map building (SLAM) [10, 13]. Some authors have even combined the EKF with fuzzy logic for a better estimation of the robot's position [1, 4].

In this paper we present the position estimation, with the help of the KF and the EKF, for an autonomous mobile robot based on Ackermann steering. Here we provide a comparison of the localization accuracy between the KF based implementation and the EKF based implementation.

### 2. Kalman filter implementation

To model the robot position, we wish to know its  $x$  and  $y$  coordinates and its orientation. These three parameters can be combined into a vector called a state variable vector. The robot uses an overhead camera to obtain the information about how far the robot has traveled to calculate its position. These measurements include a component of error. If trigonometry is used to calculate the robot's position it can have a large error and can change significantly from frame to frame depending on the measurement at the time. The Kalman filter is a smarter way to integrate measurement data into an estimate by recognising that measurements are noisy and that sometimes they should be ignored or have only a small effect on the state estimate. It smooths out the effects of noise in the state variable being estimated by incorporating more information from reliable data than from unreliable data. The user can tell the Kalman filter how much noise there is in the system and it calculates an estimate of the position taking the noise into account.

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**Keywords:** electron beam, deflecting system, simulation

It is less likely to incorporate one bad measurement if the current position confidence is high.

For the KF implementation we have used the kinematic model of an autonomous mobile robot based on Ackermann steering (see figure 1). The mobile robot is supposed to move in a 2D coordinate system.

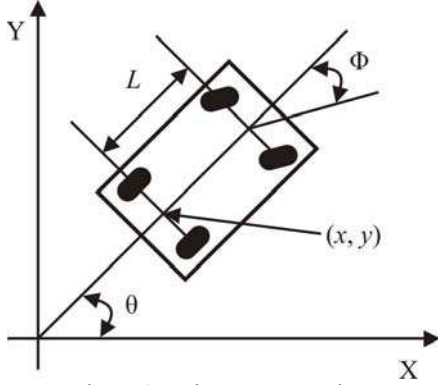


Figure 1. Robot representation.

The kinematic equations for the mobile robot:

$$\dot{x} = V \cdot \cos \theta, \quad (1)$$

$$\dot{y} = V \cdot \sin \theta, \quad (2)$$

$$\dot{\theta} = \frac{V \cdot \tan \Phi}{L}. \quad (3)$$

Where  $V$  is the velocity of the robot,  $L$  represents the distance between the rear axel and front one, and  $\Phi$  is the steering angle.

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} x_k + \Delta t \cdot V \cdot \cos \theta \\ y_k + \Delta t \cdot V \cdot \sin \theta \\ \frac{\Delta t \cdot V \cdot \tan \Phi}{L} \end{bmatrix}, \quad (4)$$

Measurements are taken from an overhead camera, and thus  $x$ ,  $y$  and  $\theta$  can be measured directly:

$$z_k = h(x_k, v_k), \quad (5)$$

where  $x_k$  is a vector containing the state variables,  $z_k$  is a vector containing the measurement variables and  $v_k$  is the observation noise.

The KF calculation steps are described below:

I. The prediction step:

$$\hat{x}_k^- = F_k \hat{x}_{k-1} + B_{k-1} u_{k-1}, \quad (6)$$

$$P_k^- = F_k P_{k-1} F_k^T + Q_{k-1}, \quad (7)$$

where  $F_k$  is the state transition model which is applied to the previous state  $\hat{x}_{k-1}$ ;  $B_{k-1}$  is the control-input model which is applied to the control vector  $u_{k-1}$ .

II. Observation:

$$\tilde{y}_k = z_k - H_k \hat{x}_k^-, \quad (8)$$

$$P_k^- = F_k P_{k-1} F_k^T + Q_{k-1}, \quad (9)$$

where  $\tilde{y}_k$  represents the innovation or measurement residual;  $H_k$  is the observation model which maps the true state space into the observed space;  $P_k^-$  represents the predicted (*a priori*) estimate covariance;  $Q_{k-1}$  is the covariance of the process noise.

III. Update:

$$K_k = P_k^- H_k^T S_k^{-1}, \quad (10)$$

$$\hat{x}_k = \hat{x}_k^- + K_k \tilde{y}_k, \quad (11)$$

$$P_k = (I - K_k H_k) P_k^-, \quad (12)$$

where  $K_k$  is the optimal kalman gain and  $S_k$  is the innovation (or residual) covariance.

### 3. The EKF implementation

Because of the nonlinear case that is encountered here we also have used an EKF that linearizes the current mean and covariance. The EKF has been applied to the same robot model as the one used for the KF case.

The EKF algorithm is described in the following:

I. The prediction step:

$$\hat{x}_k^- = f(\hat{x}_{k-1}^-, u_{k-1}, \begin{matrix} 0 \\ w_k=0 \end{matrix}), \quad (13)$$

$$P_k^- = F_k P_{k-1} F_k^T + W_k Q_{k-1} W_k^T, \quad (14)$$

where  $W_k$  is the Jacobian matrix of partial derivatives of  $f$  with respect to  $w_k$  and  $F_k$  is the Jacobian matrix of partial derivatives of  $f$  with respect to  $x_k$ .

Now we need to calculate the Jacobians  $F_k$  and  $W_k$ :

$$F_k = \begin{bmatrix} 1 & 0 & -v \cdot \sin \theta \\ 0 & 1 & v \cdot \cos \theta \\ 0 & 0 & 1 \end{bmatrix}, \quad W_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (15)$$

II. Observation and update:

In this step we need to compute the Kalman gain  $K_k$ :

$$K_k = P_k^- J_k^T (J_k P_k^- J_k^T + V_k R_k V_k^T)^{-1}. \quad (16)$$

Now, we need to compute the Jacobians  $J_k$  and  $V_k$ :

$$J_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, V_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (17)$$

where  $J_k$  is the Jacobian matrix of partial derivatives of  $h$  with respect to  $x_k$  and  $V_k$  is the Jacobian matrix of partial derivatives of  $h$  with respect to  $v_k$ .

The measurement update equations are:

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H), \quad (18)$$

$$P_k = (I - K_k J_k) P_k^-. \quad (19)$$

#### 4. Simulation results

In this section of the paper we compare the results obtained by simulating the KF case, with the ones obtained from the EKF case. For this purpose both filters were implemented in Matlab. We have considered a very simple case: a robot that follows a path obtained from the system model. In the figures it is presented the estimated path of the robot compared to the real path.

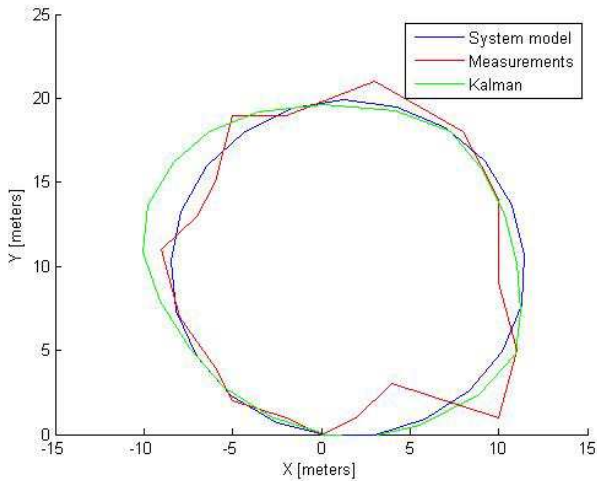


Figure 2. Trajectory estimation with the KF.

In figure 2 it is shown the path estimated with the help of the KF, and in figure 3 is presented the path estimated with the help of the EKF.

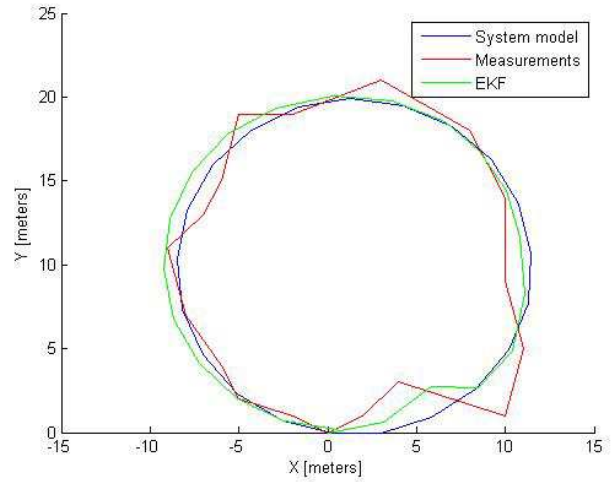


Figure 3. Trajectory estimation with the EKF.

Both filters performed very well, but it can be seen that the EKF has performed better and as a result the estimated path with the help of this filter is closer to the real path. To better evaluate the performance of the two filters, we plotted the actual states and the estimated ones for the entire simulation. For this purpose we have plotted the estimated data for the X axis and compared it to the real data (see figure 4).

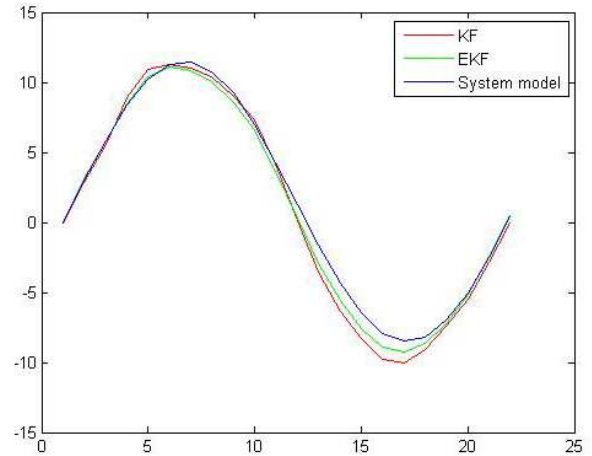


Figure 4. Illustration of using the EKF and KF to estimate the position on the X axis.

We also plotted the estimated data for the Y axis (see figure 5). In both plots it can be seen clearly that the EKF (green line) predicted path is closer to the real path (blue line). At first the KF (red line) performed almost as well as the EKF, but as the simulation continued the errors of the KF were bigger than the ones of the EKF. The difference between the performance of the two filters can be seen above.

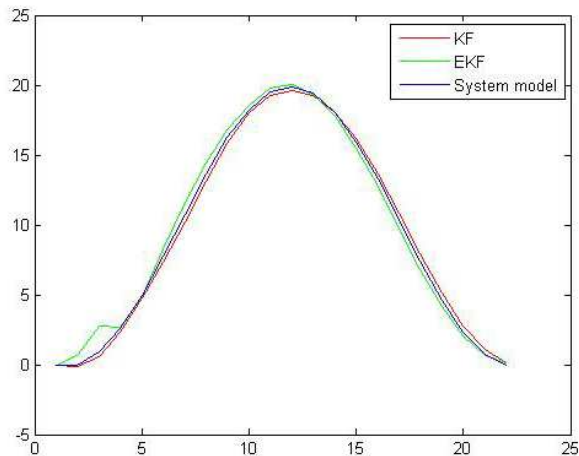


Figure 5. Illustration of using the EKF and KF to estimate the position on the Y axis.

## 5. Conclusions

In this paper the standard Kalman filter and one of the main variations of this filter, the extended Kalman filter, are used for the position estimation of an autonomous mobile robot based on Ackermann steering. Comparing the simulation results show that the performance of the EKF is consistently better than the KF. In the future we will try to combine the fuzzy logic with one of the filters presented in this paper for a better position estimation. Furthermore, we will use the EKF in a visual based SLAM system, where the camera calibration and correct feature detection play a vital role in solving the data association problem.

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