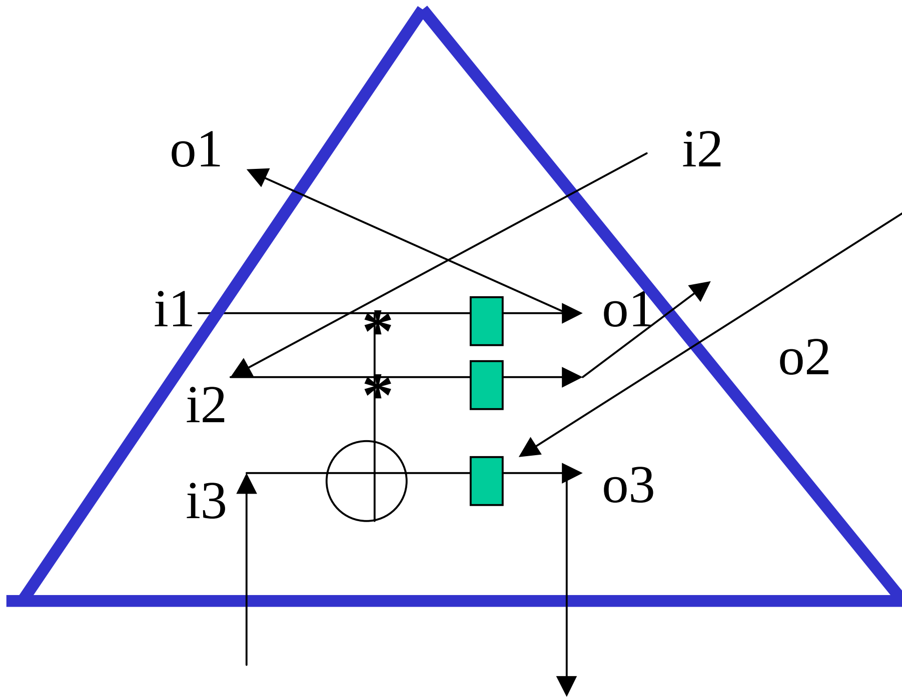
Solving correctly seven problems is sufficient for a grade of A

- 1. The more you solve the better.
- 2. Try to verify each your solution.
- 3. Try to explain what you do in English. Do not worry about grammar or spelling. Try just to explain your intentions in full sentences.
- 4. A’ means negation of A for all signals in the slides below.

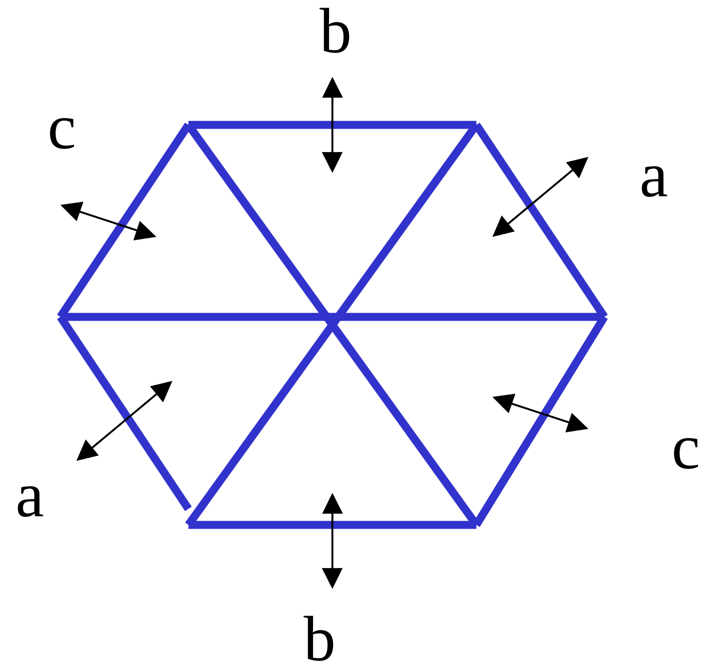
Problem 1

Given is the cell of a cellular automaton

Grey rectangles are D flip-flops, clocks not shown.



The structure of cell connections is shown below. There are three cells as in left and three cells rotated clockwise by 180 degree



A). Analyze behavior of the system in which all cells are in state 0. The state is a natural number corresponding to binary signals in order o1,o2,o3, with o3 as the least significant bit.

B). Analyze first two transitions of the system in which all cells are in state 7.

C). Try to guess the initial state that will lead to the longest cycle. Show your work.

D) Can you generalize these results to arbitrary size CA of this type of cells and connection structure?

Problem 2

- A) What is the inverse gate of the Fredkin Gate?
- B) Prove that the gate that you found is inverse to Fredkin gate using the method of graphical transformation in quantum notation.
- C) Do the same using truth tables of the gates.

Problem 3

A. Give an example of a gate that is reversible and conservative

B. Give an example of a gate that is reversible and not conservative.

C. Give an example of a gate that is conservative but not reversible.

Prove all your results. Verify.

Problem 4

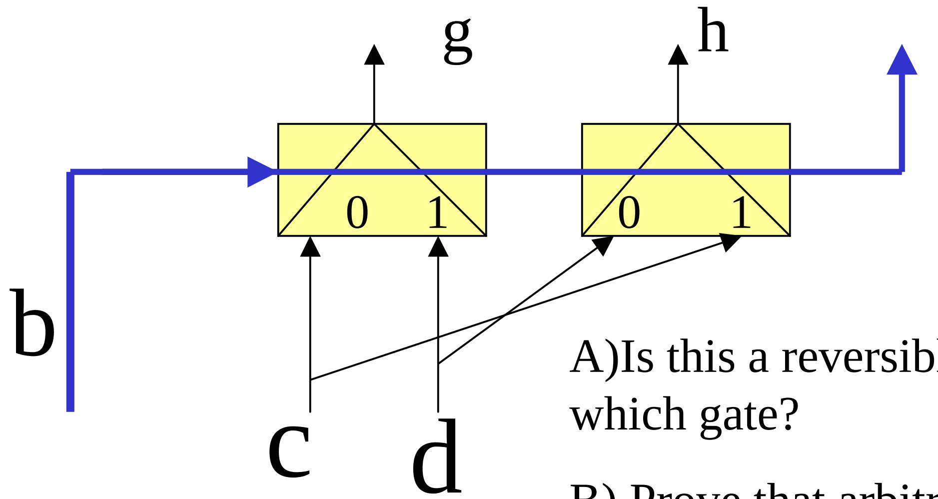
- A) Realize the Fredkin Gate using Billiard Ball Model.
- B) Realize the Swap Gate using the Billiard Ball Model

Problem 5

- A) Define what is Kronecker (Functional) Decision Diagram
- B) Derive such a diagram for functions $F=A(C\oplus B) \oplus CD$, $G= C\oplus B$ sharing as much of the diagram for both functions together.
- C) Convert the diagram to a circuit with Inverter, Toffoli and Feynman gates. Show all constant (if any) and garbage (if any) signals.
- D) Add the mirror and spy circuits in a standard way. Discuss the garbage in the new circuit.

Problem 6

Given is cell with 3 inputs and 3 outputs.

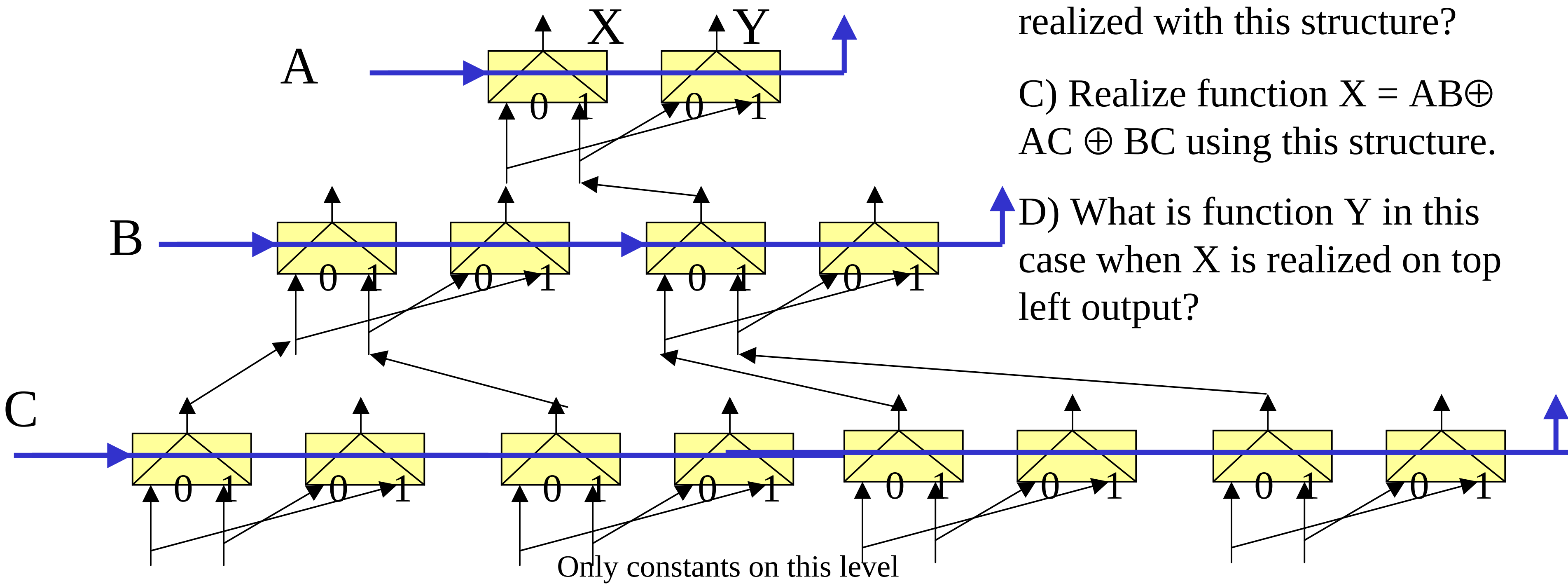


A)Is this a reversible gate? If yes, which gate?

B) Prove that arbitrary symmetric function X(A,B,C) can be realized with this structure?

C) Realize function X = AB⊕ AC ⊕ BC using this structure.

D) What is function Y in this case when X is realized on top left output?



Problem 7

- Realize Converter from Gray code to Binary Natural Code using only reversible gates. Try to minimize Garbage.

Problem 8

- Discuss the importance of work of Bennett and Landauer related to reversible logic.

Problem 9

- A) Realize ESOP for the function shown in Kmap. Minimize the number of gates and inputs to gates.
- B) Write an equivalent Positive Polarity Reed-Muller form for this function.
- C) Draw a reversible cascade in quantum notation in which function F(g,b,c,d) is one of outputs, and other outputs are arbitrary. Decrease the width of this cascade. Minimize Garbage.

gb \ cd	00	01	11	10
00	(0) 1	(1) 1	(3) 0	(2) 0
01	(4) 1	(5) 0	(7) 0	(6) 1
11	(12) 0	(13) 1	(15) 1	(14) 0
10	(8) 0	(9) 0	(11) 1	(10) 1

Problem 10

cba \ YZ	00	01	11	10
000	0	0	-	-
001	0	0	-	-
011	0	-	-	0
010	1	-	-	1
110	-	1	1	-
111	-	1	1	-
101	-	-	1	1
100	-	-	0	0

A) Groups shown in the map correspond to prime terms of an SOP. Draw a BDD of this function.

B) Groups shown in the map correspond to product terms of an ESOP. Draw a KFDD of this function that uses only Positive Davio gates.

C) Write the Positive Polarity Reed Muller form and ESOP expression.

Problem 11

A) How to recognize a symmetric function in a Karnaugh Map?

B) How to recognize a linear function in a Karnaugh Map?

C) Give an example of Kmap of 4 variable function that is both linear and symmetric?

D) Can you give an example of function that is linear but not symmetric? If not, why?

E) Realize function $S^{0,1,3,8}(A,B,C,D,E)$ in the most efficient way.

WX \ YZ	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Problem 12

- Given is a circuit described by equations:
  - $A = ab+cd'+aef$
  - $B = ab + cef + (d \oplus f)$
  - $C = acd' + (a' fg + (bd') * (c + fd))$

A) Draw a reversible realization of this circuit with arbitrary gates.

B) Prove that your circuit is reversible

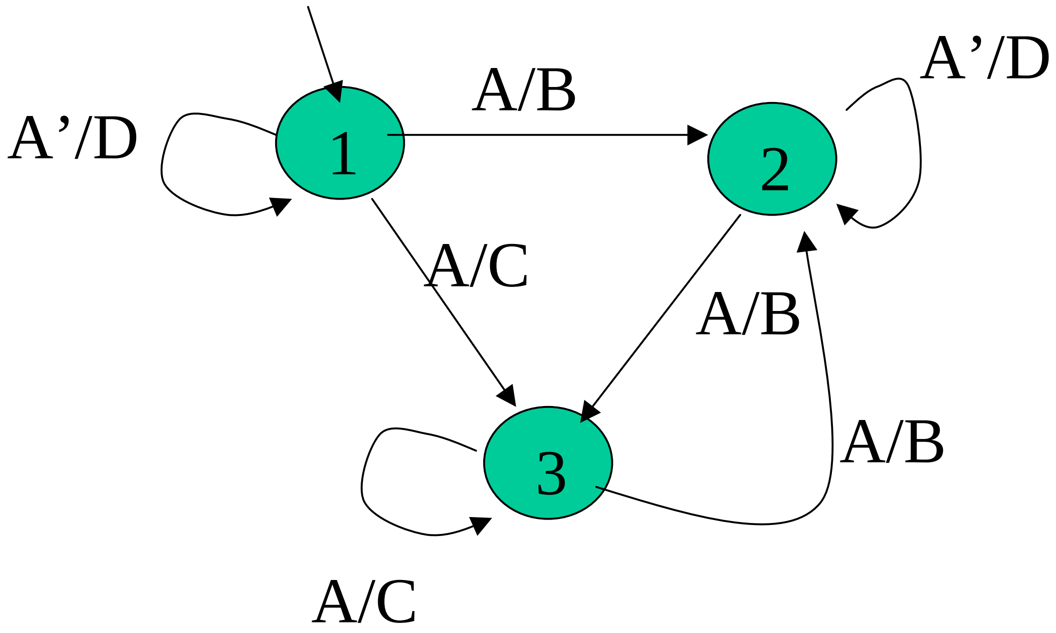
C) Find an inverse circuit

Problem 13

Realize Margolus gate with a minimum number of Toffoli and Feynman gates (and inverters).

Problem 14

- A) Convert non-deterministic machine to a deterministic one.
- B) Realize the non-deterministic (Mealy) machine using standard one-hot code realization shown in class.



Problem 15

A ternary function with inputs Z and X and outputs U and V that is specified by the map

Z \ X	0	1	2
0	0,0	0,1	0,2
1	1,1	1,2	1,0
2	2,2	2,0	2,1

U,V

A) Is this function symmetric?

B) Is this function reversible?

C) Draw this function using the minimum number of ternary reversible gates.

D) Using this function, draw a realization of a 3-qubit ternary linear function with the minimal number of gates.

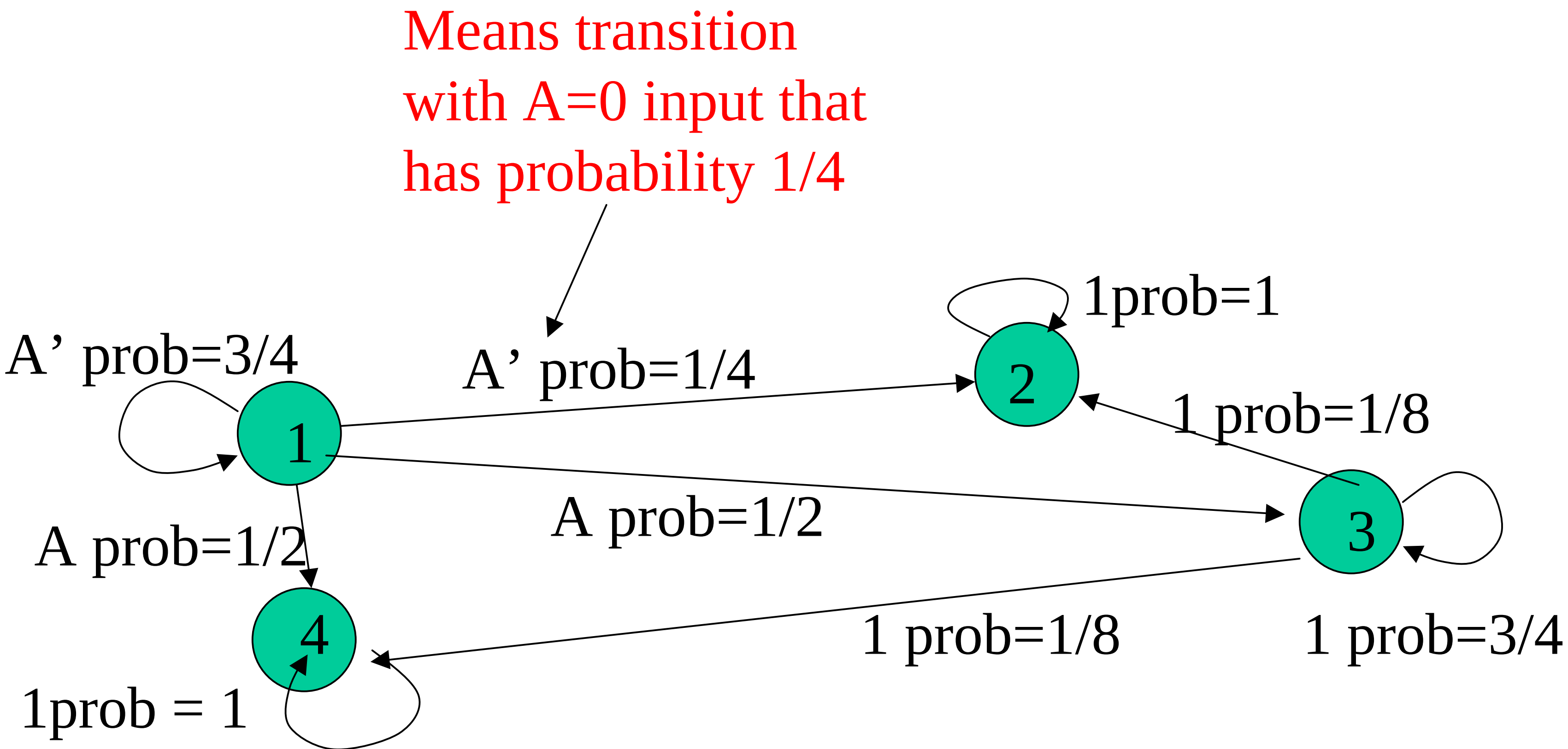
E) Extend the concept of binary controlled gate to the concept of ternary controlled gate and show two examples of such gates.

Problem 16

- Realize a ternary swap gate with arbitrary gates that are generalizations of binary gates. Prove that it really works as a swap of arbitrary ternary signals.

Problem 17

- Assuming that you have a generator of probability 1/2 and arbitrary logic gates and flip-flops, realize the following probabilistic state machine.



Problem 18

- Given is a graph.

- Show and explain a backtracking algorithm that finds the exact minimal coloring to this graph.
- Or, if you do not know the backtracking tree-searching algorithm show any other algorithm to find the minimum coloring for arbitrary graphs.
- You may use trees to explain operation of your algorithm.