Turing Machine

slides of Prof. K. J. Hintz used

Turing Machine

- An FSM Cannot Compute a Number Comprised of an Infinite Number of Digits Since It Itself Is Finite.
- A Finite Method Can Be Developed to Compute an Infinite Length Number
- The Method Can Be Implemented by Reading and Writing Intermediate Results from/to an Assumed Infinite Memory

Halting Problem

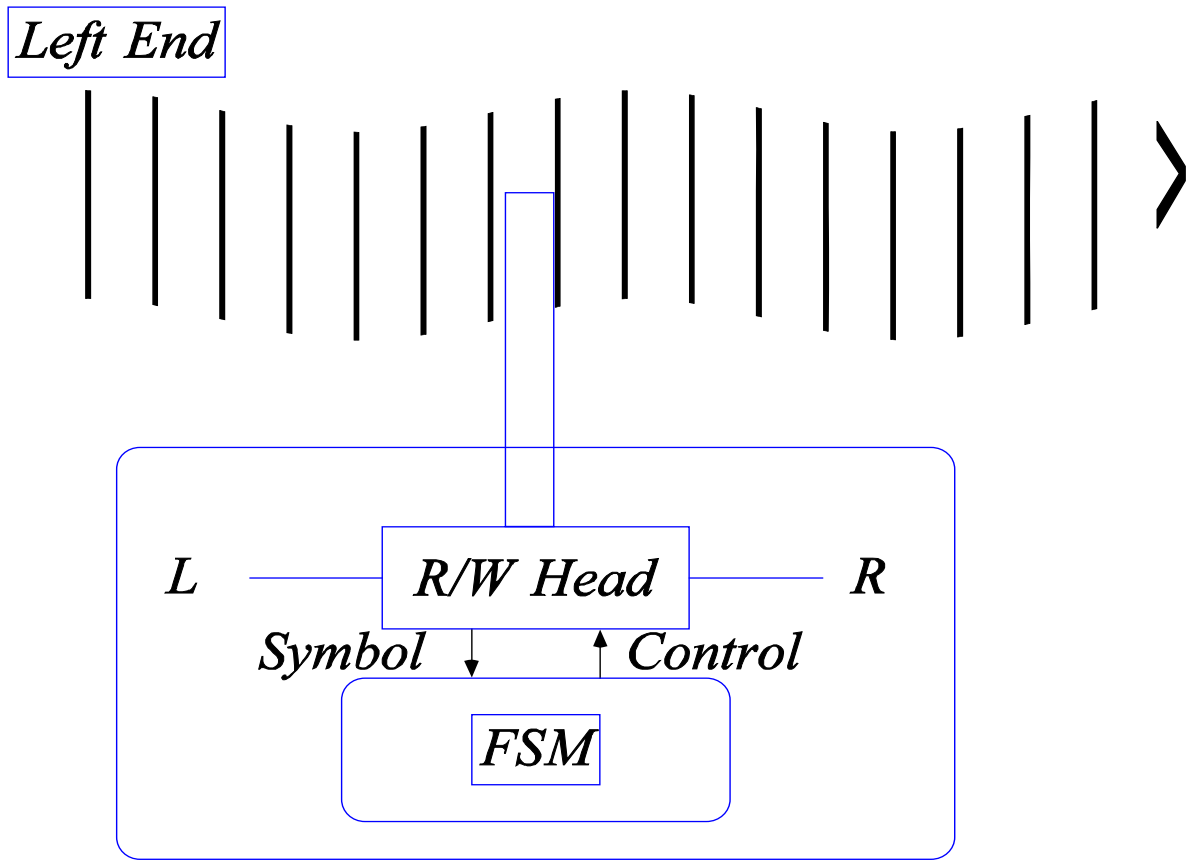
- Difficulty with the Turing Machine
  - Not guaranteed to terminate
  - Don’t know when it will terminate
- Still an unsolved problem
  - Is there a method for analyzing a Turing machine to determine if and when it will terminate its computation?

Turing Machine

- Devised by Alan Turing, 1937 paper*
- Definition of Computable
- Some Problems are Not Computable

* Turing, A. M., "On Computable Numbers, with an Application to the Entscheidungsproblem", *Proceedings of the London Mathematical Society,* Series 2, 42:230-265 and 43:544-546, 1937.

Physical Model of TM



Properties

- Infinitely Long Readable/Writable Tape With Fixed Left End
- Moveable Read/Write Tape Head
  - Only read/write to current position
- Finite State Machine for Control
  - Movement of tape head +/- 1 tape position only
  - Symbols written to tape

Properties

- String on Tape
  - Left End of String at Left End of Tape
  - Tape Head Is Initially at Right End of String.

Left end of Tape *Tape Head Initial Position*

TM Alternatives

- Multiple Heads
- Infinite Tape in Both Directions
- Tape Moves, Head Stationary
- Increased Number of Symbols

None of These Changes Affect the Basic Computability of Turing Machines

TM Formal Definition

- A Turing Machine is a Quadruple

$$M_T = (\mathbf{S}, \Sigma, s, \delta)$$

S Finite set of states

Σ $\{$ Tape Symbols $\notin \{ L, R \} \} \cup \{ \# \}$

s initial state

δ Deterministic transition function

$$\delta : S \times \Sigma \rightarrow (S \cup h) \times (\Sigma \cup \{ L, R \})$$

TM Configuration

- The TM Configuration is a quadruple

$$C_T = (s, t_L, t, t_R)$$

$$\in (\mathbf{S} \cup \{ h \}) \times \Sigma^* \times \Sigma \times (\Sigma^* (\Sigma - \{ \# \}) \cup \{ \Lambda \})$$

s	Current State
t_L	Sequence of symbols to the left of the tape head $\in \Sigma^*$
t	Current read symbol under the head
t_R	Sequence of symbols to the right of the tape head

TM Configuration

- A Shorthand Notation

(current state,

string with current head position underlined)

e.g.,

$$(q_2 , abb\underline{a}ab)$$

TM Delta Function

$$\delta : \mathbf{S} \times \Sigma \rightarrow \left(\mathbf{S} \cup \{ h \} \right) \times \left(\Sigma \cup \{ \mathrm{L}, \mathrm{R} \} \right)$$

Present State	*Read Present Symbol*	*Next State & Symbol*
q1	a	(p, b)
q2	#	(p, R)

If read an *a*, write a *b*.

If read a *blank (#)*, read-only, and move right one.

Two TM Examples

- Quantity Doubler:
  - State Table: head movement and write are done in separate states
  - FSM: simultaneous write/move
- Odd Detector
  - Head movement and write are done simultaneously

TM Example, 2X

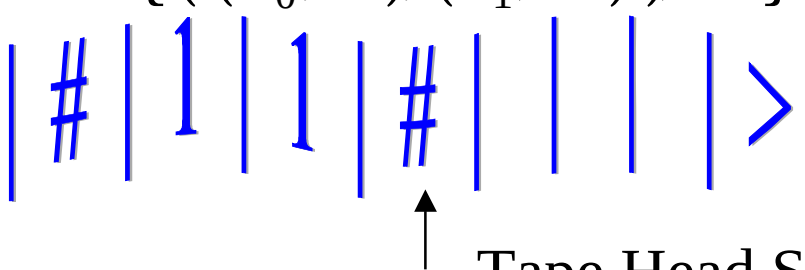
$$M_{T2x} = (\mathbf{S}, \Sigma, s, \delta)$$

$$\mathbf{S} = \{0,1,2,3,4,5\}$$

$$\Sigma = \{\#,1\}$$

$$s = s_0$$

$$\delta = \{((s_0,\#),(s_1,L)),\cdots\}$$



Tape Head Starting Position

TM Example, 2X

- Transition Function

PS	Input #	Input 1	Comment
s_0	s_1/L	s_0/L	
s_1	halt	s_2/#	found a 1 to left, replace with #
s_2	s_3/R	s_3/R	

TM Example, 2X

PS	Input #	Input 1	Comment
s_3	s_4/1	s_3/R	Move to right until at right-most end of string of 1's
s_4		s_5/R	
s_5	s_0/1	s_5/R	

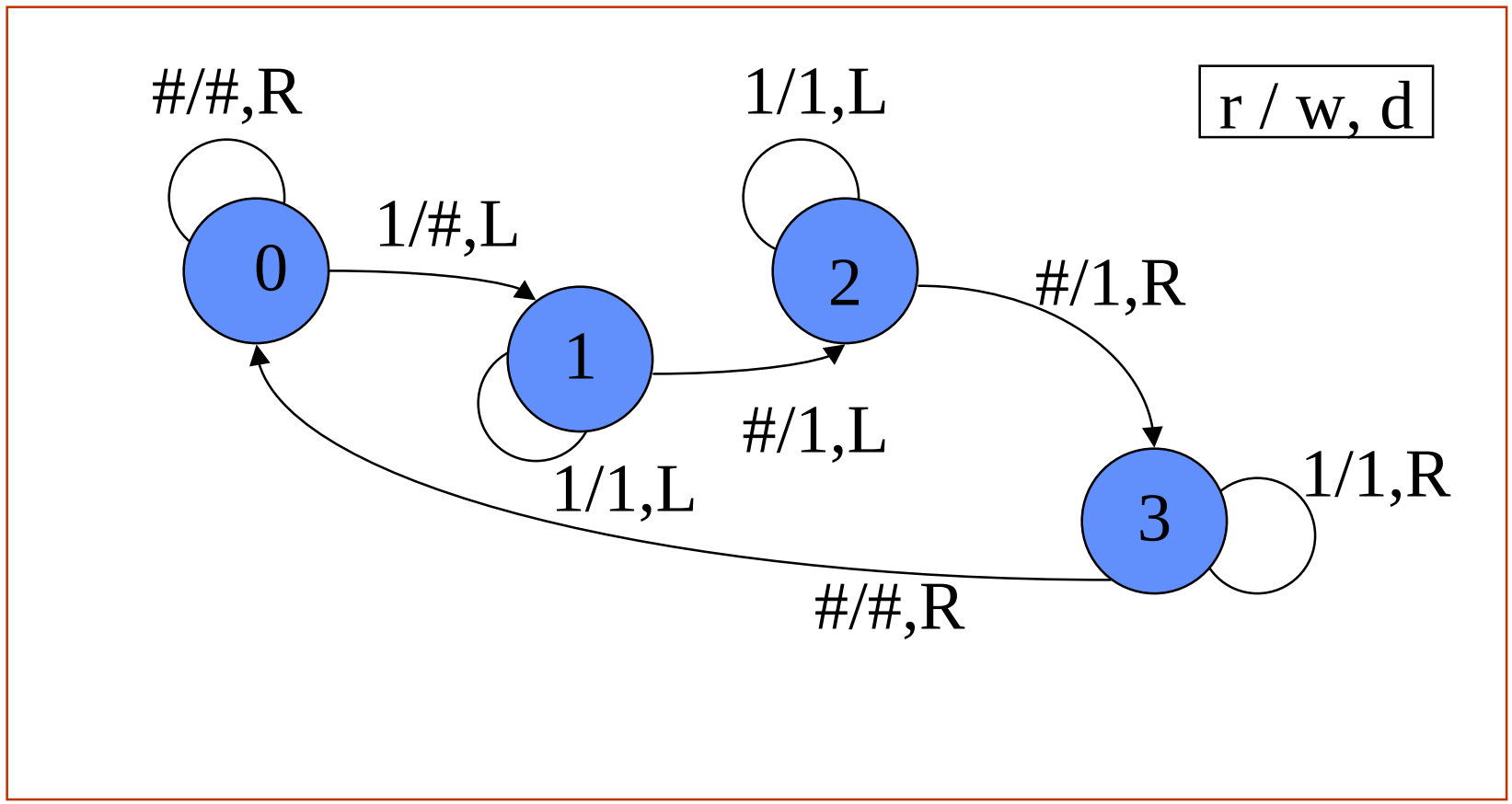
TM Example, 2X

#	1	1	#		s_0	#	1	#	1	1	#		s_0
#	1	1	#		s_1	#	1	#	1	1	#		s_1
#	1	#	#		s_2	#	#	#	1	1	#		s_2
#	1	#	#		s_3	#	#	#	1	1	#		s_3
#	1	#	1	#	s_4	#	#	1	1	1	#		s_4
#	1	#	1	#	s_5	#	#	1	1	1	#		s_5
#	1	#	1	1	# s_0	#	#	1	1	1	#		s_5
#	1	#	1	1	# s_0	#	#	1	1	1	#		s_5

TM Example, 2X

#	#	1	1	1	$\underline{1}$	s_5
#	#	1	1	1	$\underline{1}$	s_0
#	#	1	1	$\underline{1}$	1	s_0
#	#	1	$\underline{1}$	1	1	s_0
#	#	$\underline{1}$	1	1	1	s_0
#	$\underline{\#}$	1	1	1	1	s_0
$\underline{\#}$	#	1	1	1	1	s_1

FSM to Control 2x TM Simultaneous Write/Move



TM Example 2, Odd Det.

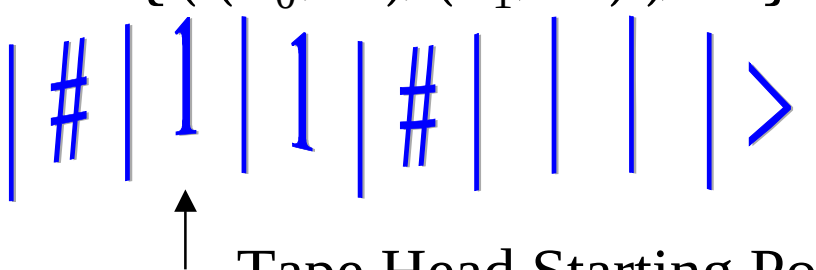
$$M_{T2x} = (\mathbf{S}, \Sigma, s, \delta)$$

$$\mathbf{S} = \{0, 1\}$$

$$\Sigma = \{\#, 1\}$$

$$s = s_0$$

$$\delta = \{((s_0, \#), (s_1, L)), \cdots\}$$



Tape Head Starting Position

TM Example 2, Odd Det.

- Transition Function

PS	Input #	Input 1	Comment
s_0	S_0/#/R	S_1/#/F	
s_1	halt	s_0/#/R	Halts if Odd