Explanation

- 1. Pick 5 problems and solve them. Solving correctly 5 problems is sufficient to obtain the grade of A.
- 2. If you have enough time, solve as many other problems as you want, but please define which are your main 5 problems.
- 3. I give partial credits for good ideas.
- 4. I subtract only some credit for numerical errors. Ideas are more important to me.
- 5. Verify your solutions.
- 6. Underline or mark otherwise your final solution to each point.

Problem 1 .

1. Given is a Truth Table

full adder

X	Y	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

1. Realize this function using BDDs. Arbitrary order of variables

2. Realize this function using Kronecker Functional Decision Diagrams. Arbitrary order of variables.

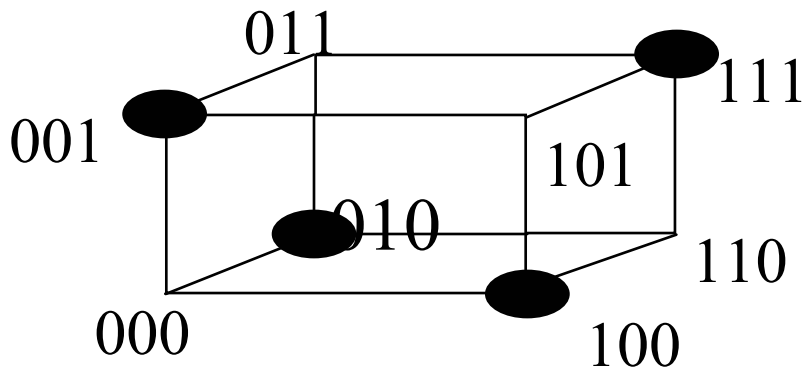
3. Show solution based on mapping BDD or KFDD to arbitrary logic gates. Draw a schematic and write expressions.

4. Realize this function using Feynman and Toffoli gates. Show and explain your design stages. Verify your solution.

In each of the above 3 sub-problems, try to minimize the total number of gates and verify your solutions.

Problem 2.

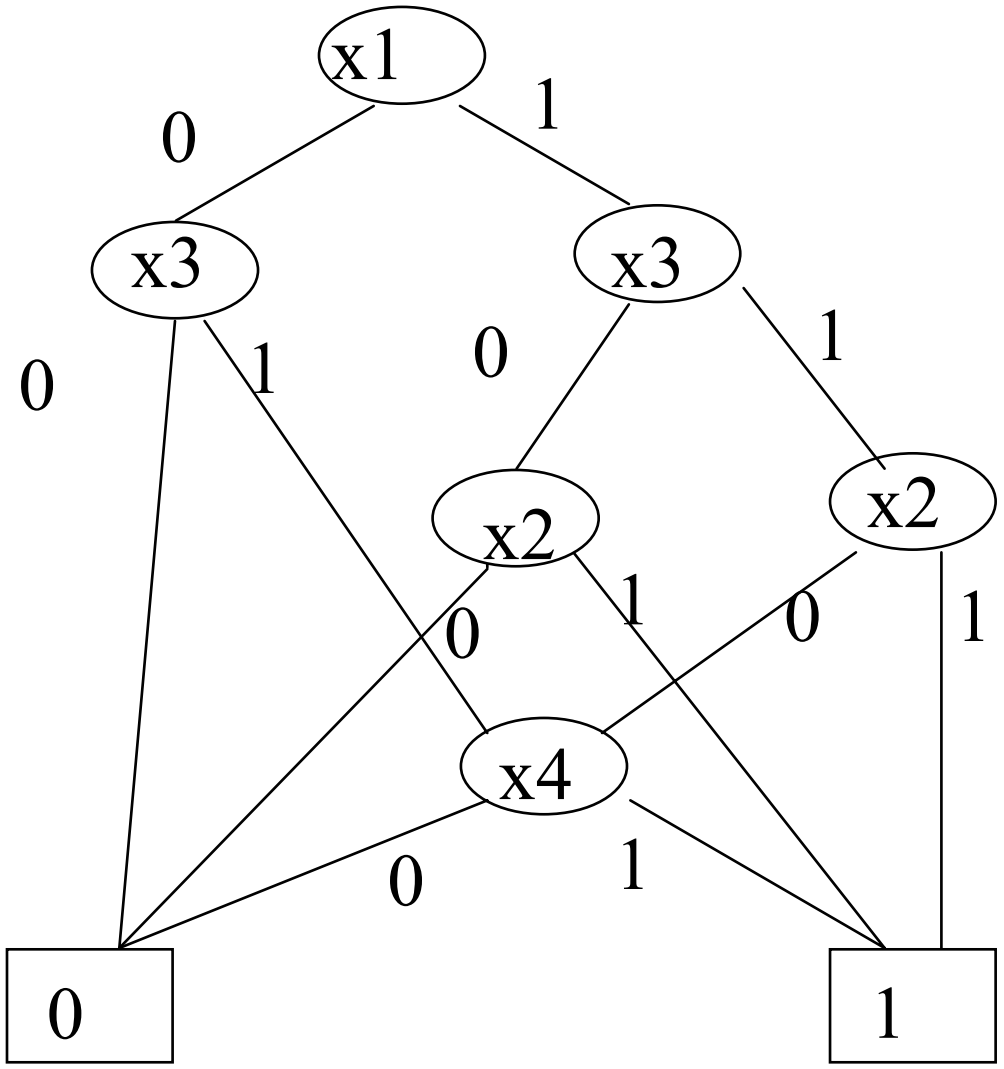
Realize this function of 3 variables as a circuit with MINIMUM number of NAND gates, each with 2 inputs. Try to prove that your solution is minimum.



Problem 3

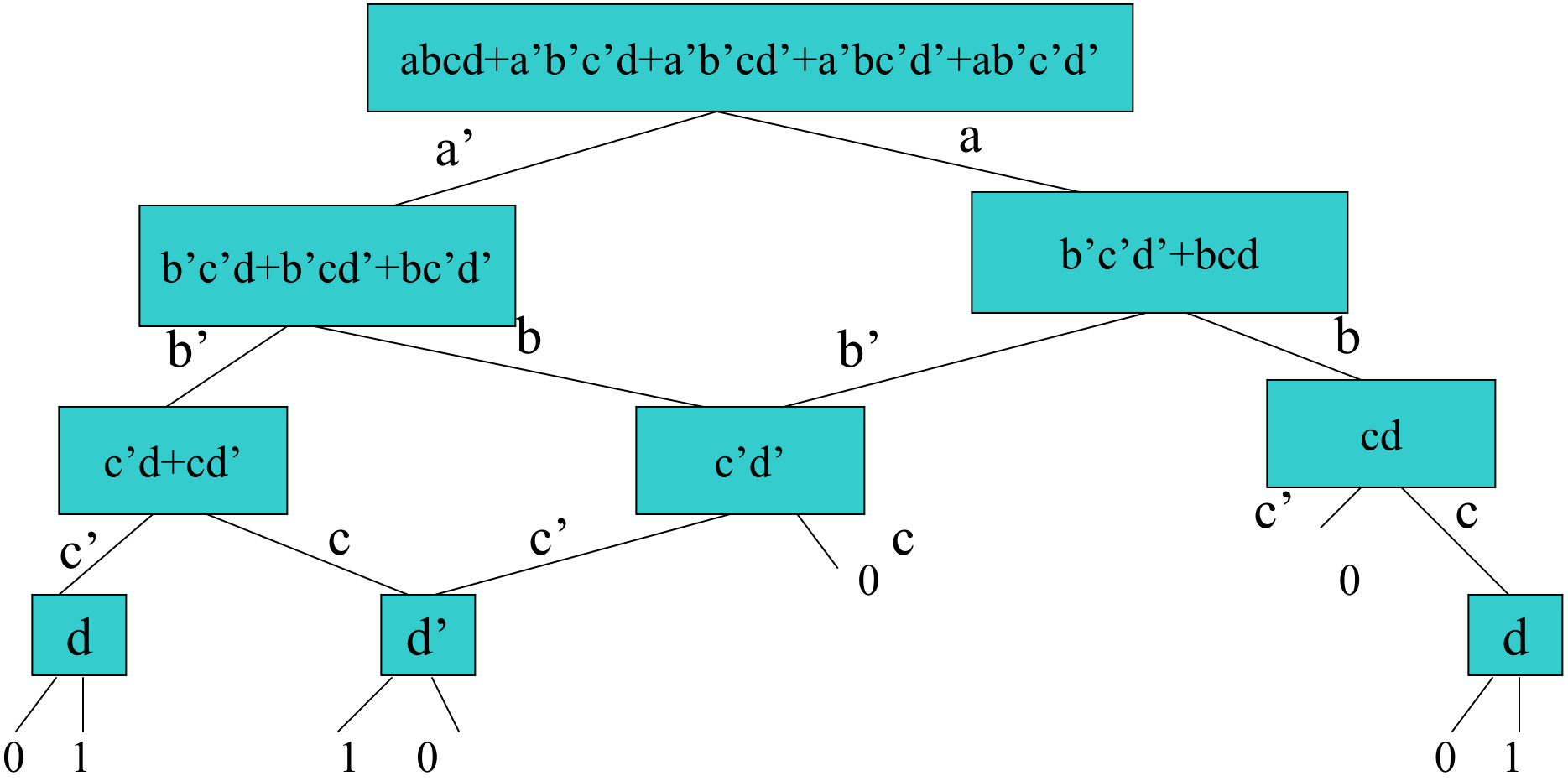
Find a better ordering for this function or show that it does not exist.

Realize the function from the improved BDD using only majority gates (majority gate is f=ab+ac+bc). Minimize the number of majority gates. Verify the solution using Kmap.



Problem 4

Realize the function expressed by this lattice diagram using only 2-input gates (arbitrary). Minimize the number of used gates. Draw a Kmap of this function and verify your solution.



Problem 5

$\frac{\partial f}{\partial x}$ is called Boolean difference of f with respect to x

$$\frac{\partial f}{\partial x} = f_x \oplus f_{\overline{x}}$$

f is sensitive to the value of x when $\frac{\partial f}{\partial x}$ *is not* 0

Find two practical applications of the Boolean Difference operation defined as above. Think about test generation and the condition for an input combination to be a test.

How to realize this operation using Cube Notation? (Cube is a product of variables and negated variables).

How to realize it using Truth Tables or Kmaps?

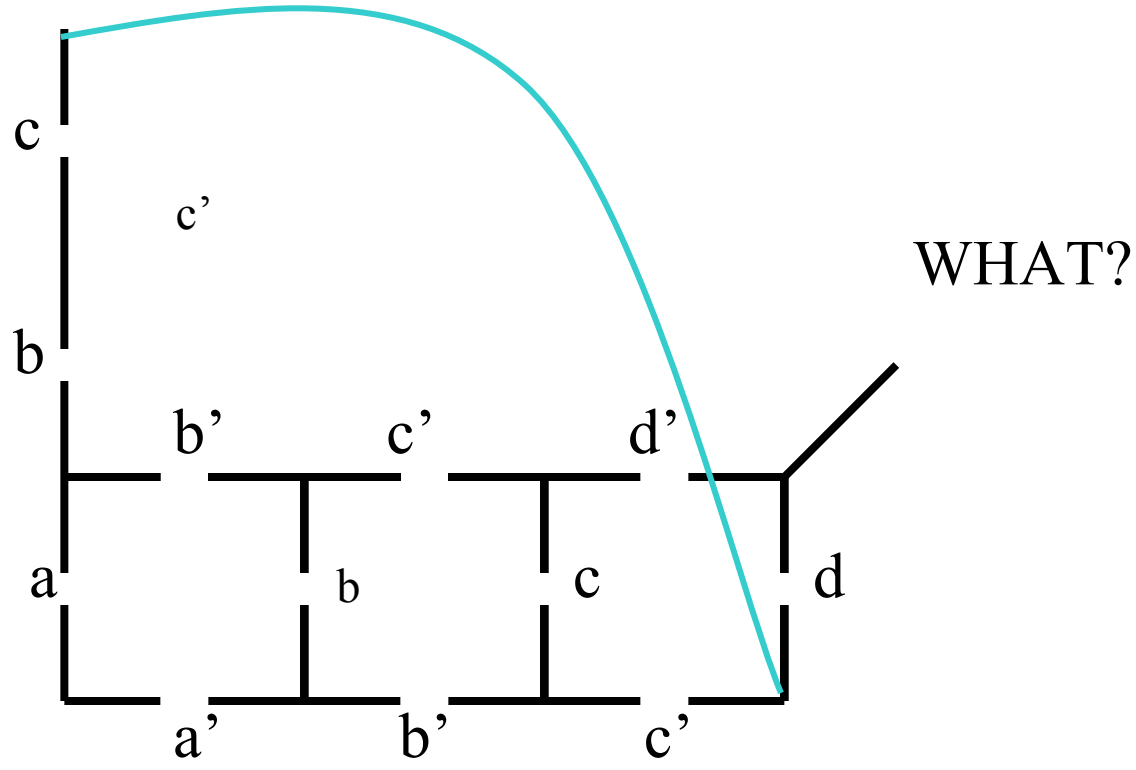
How to realize it using BDDs?

How to realize it using Functional Decision Diagrams (FDDs)?

PROBLEM 6

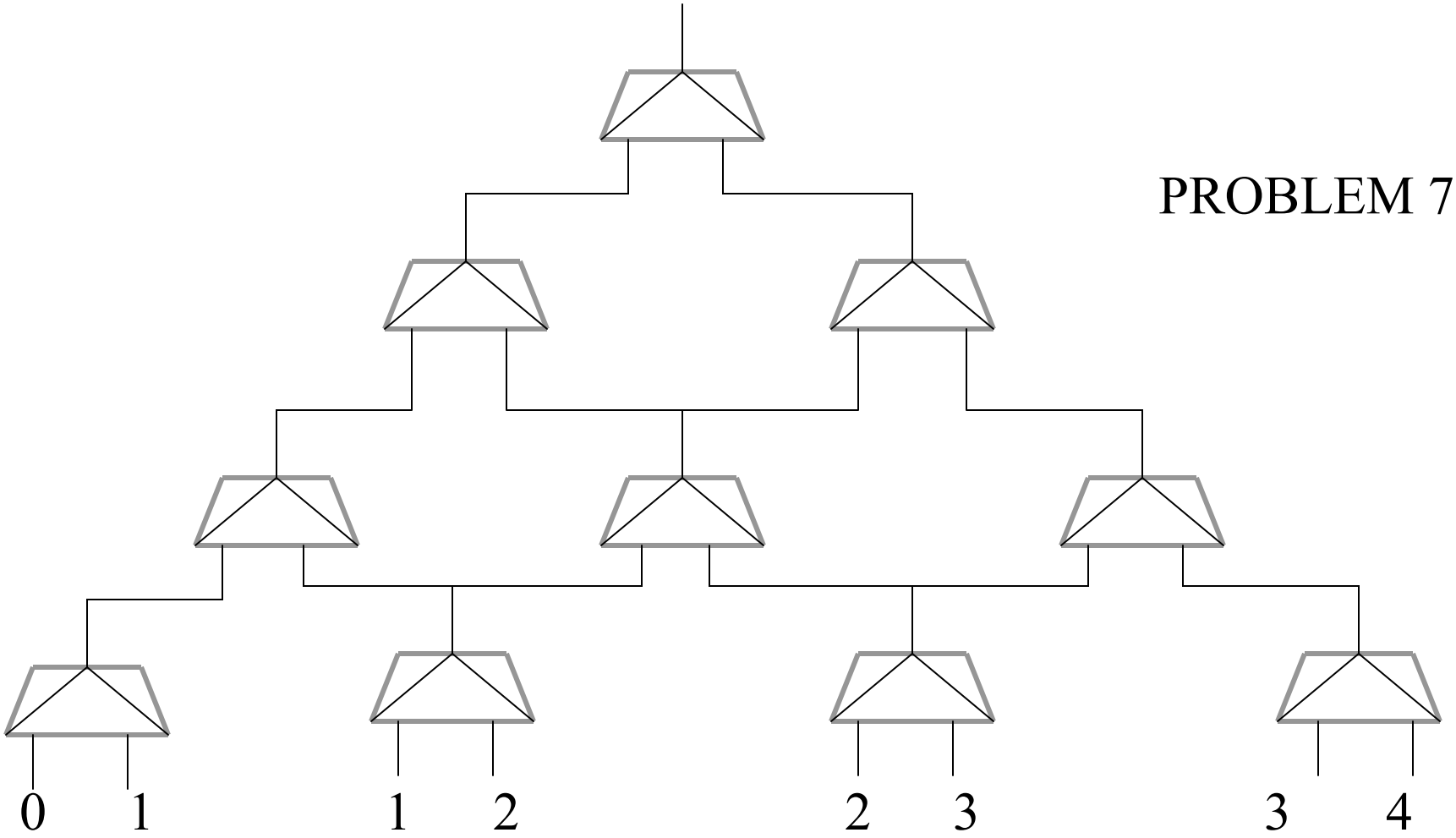
1. What is the function realized here? Show Kmap and analyze all paths.

2. Realize this function with only multiplexers.



The symmetric function can be mapped in to the lattice of multiplexers as shown in the figure below. Can you prove that by replacing multiplexers with ARBITRARY other gates we will be able still to realize arbitrary symmetric function. Control variables of multiplexers are not shown here.

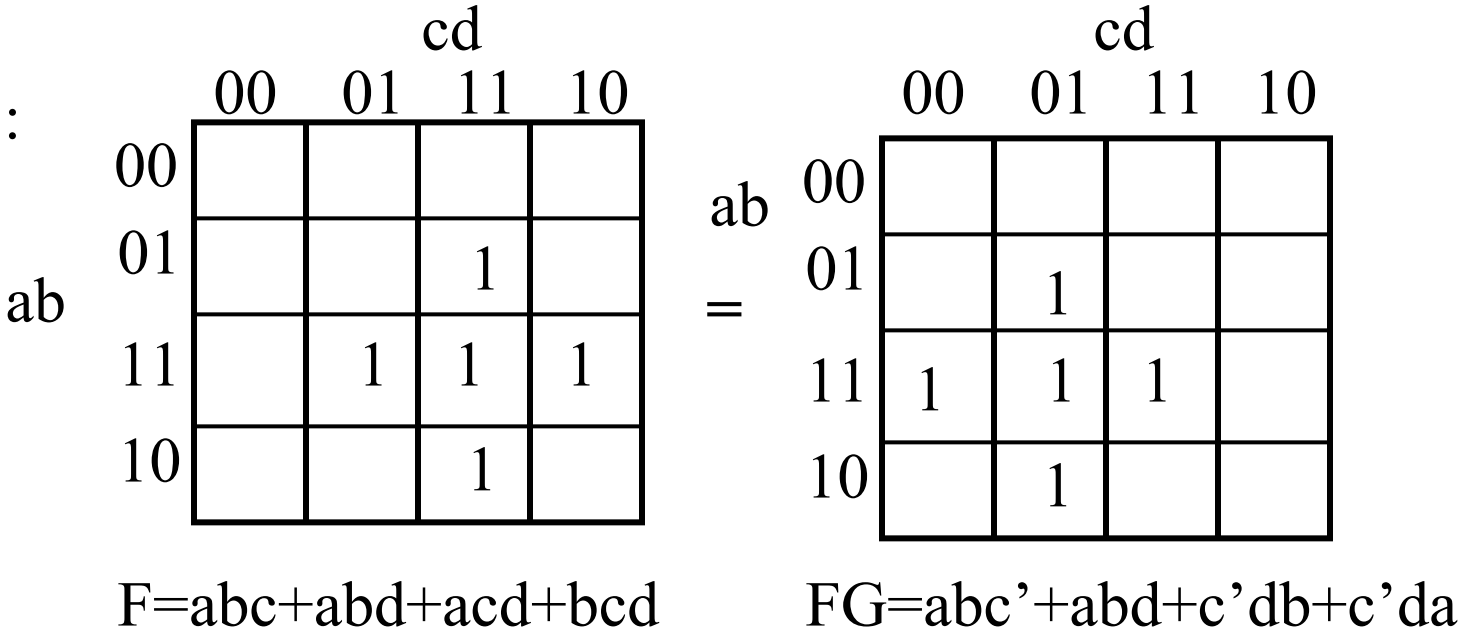
PROBLEM 7



Problem 8

Mirror image function of any symmetric function is symmetric for same negated variables for which original function is symmetric.

For example :



F=abc+abd+acd+bcd

FG=abc'+abd+c'db+c'da

both are symmetric functions

Use this observation to create a more general definition of symmetry and a regular structure to realize all generalized symmetric functions according to your definition. Show solution to function FG, using only multiplexers and negations.

Problem 10. Definitions and ideas.

- 1. Give definitions of prime implicant, prime implicate, essential prime implicant
- 2. Give a definition of a maximum clique of a graph.
- 3. Define Shannon and Davio expansions and show gates for them.
- 4. What the applications of gates and expansions from point 3?
- 5. What are vacuous variables. Give examples.
- 6. Give example of Pareto Minimization.
- 7. Discuss satisfiability versus complementation versus tautology. How you can use a satisfiability solver to check tautology. Show how to do complementation algorithmically using SOP and POS formulas, not Kmaps.

Problem 11. Algorithms.

- 1. List all known to you methods to solve the covering problem.
- 2. What is the difference of Binate and Unate Covering
- 3. Boolean equations and Petrick Functions. Show examples.
- 4. Color the graph from Figure A and prove that this coloring is minimal
- 5. Assuming that this graph is a compatibility graph, draw the incompatibility graph and its clique covering.

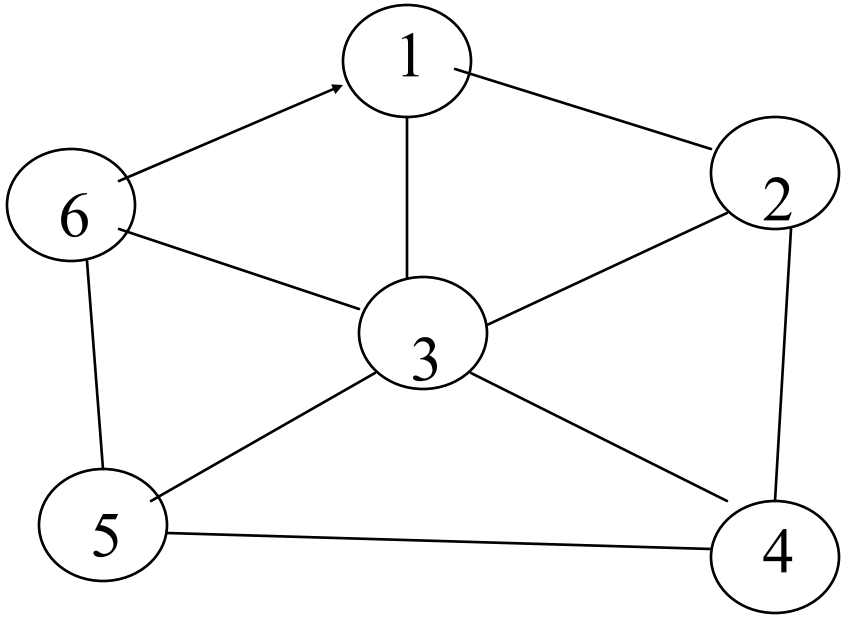


Figure A

Problem 12

1. Find the minimal ESOP circuit.
2. Draw its schematic
3. Verify graphically that your solution is correct.

Use the concept of incomplete tautology.

ab \ cd	00	01	11	10
00		1	0	1
01			1	
11		1	0	1
10			1	

All other cells are don't cares