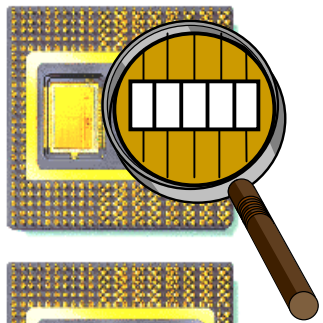
Register Allocation (via graph coloring)

Lecture 25

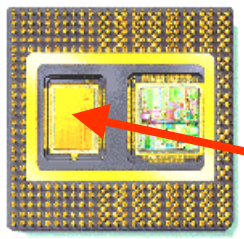
Lecture Outline

- Memory Hierarchy Management
- Register Allocation
  - Register interference graph
  - Graph coloring heuristics
  - Spilling
- Cache Management

The Memory Hierarchy



Registers 1 cycle 256-8000 bytes



Cache 3 cycles 256k-1M

Main memory 20-100 cycles 32M-1G

Disk 0.5-5M cycles 4G-1T

Managing the Memory Hierarchy

- Programs are written as if there are only two kinds of memory: main memory and disk

- Programmer is responsible for moving data from disk to memory (e.g., file I/O)
- Hardware is responsible for moving data between memory and caches
- Compiler is responsible for moving data between memory and registers

Current Trends

- Cache and register sizes are growing slowly
- Processor speed improves faster than memory speed and disk speed
  - The cost of a cache miss is growing
  - The widening gap is bridged with more caches
- It is very important to:
  - Manage registers properly
  - Manage caches properly
- Compilers are good at managing registers

The Register Allocation Problem

- Recall that intermediate code uses as many temporaries as necessary
  - This complicates final translation to assembly
  - But simplifies code generation and optimization
  - Typical intermediate code uses too many temporaries
- The register allocation problem:
  - Rewrite the intermediate code to use fewer temporaries than there are machine registers
  - Method: assign more temporaries to a register
    - But without changing the program behavior

History

- Register allocation is as old as intermediate code
- Register allocation was used in the original FORTRAN compiler in the ‘50s
  - Very crude algorithms
- A breakthrough was not achieved until 1980 when Chaitin invented a register allocation scheme based on graph coloring
  - Relatively simple, global and works well in practice

An Example

- Consider the program

  $a := c + d$

  $e := a + b$

  $f := e - 1$

  - with the assumption that a and e die after use
- Temporary a can be "reused" after $e := a + b$
- Same with temporary e
- Can allocate a, e, and f all to one register (r_1):

  $r_1 := r_2 + r_3$

  $r_1 := r_1 + r_4$

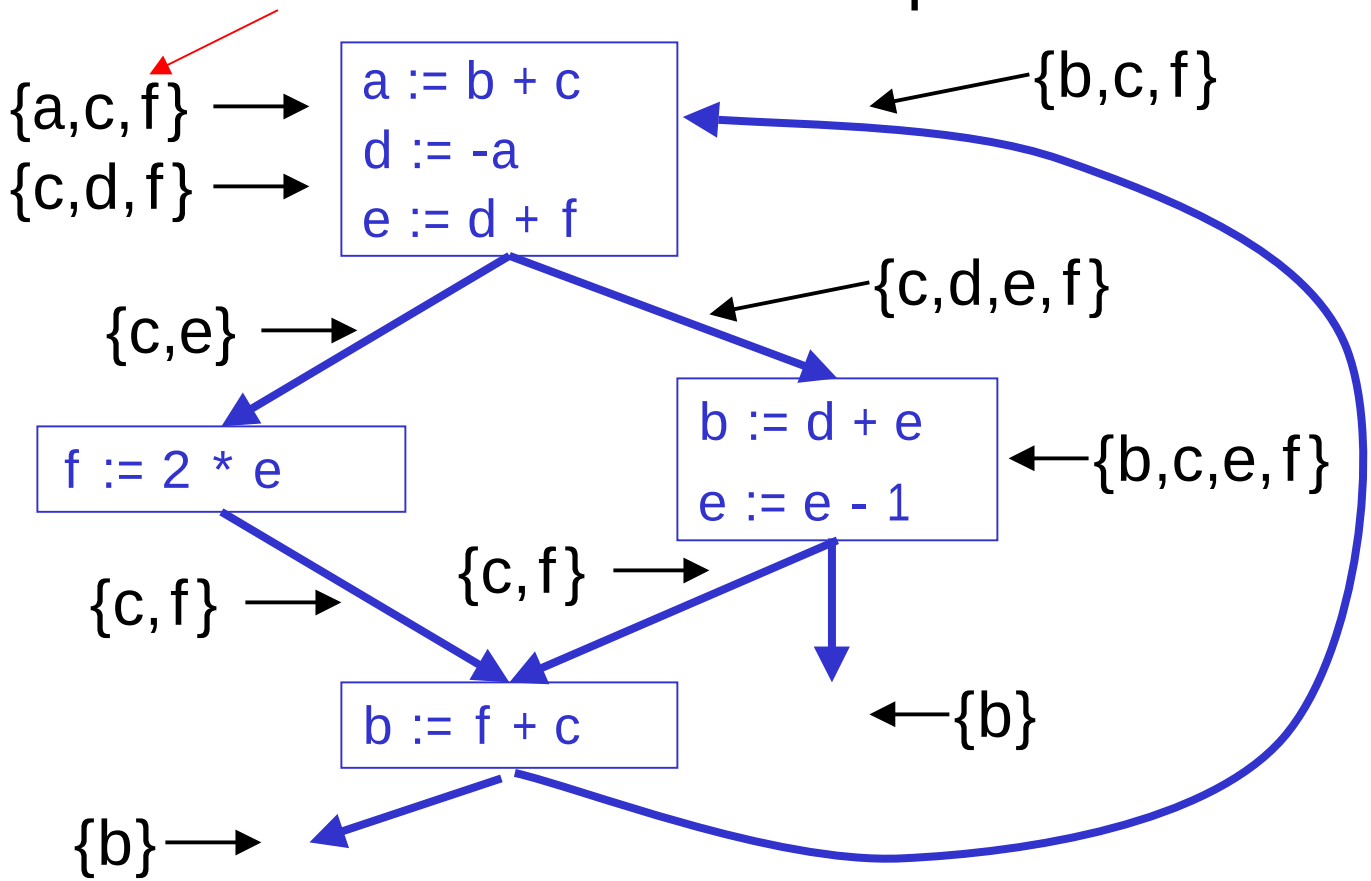
  $r_1 := r_1 - 1$

Basic Register Allocation Idea

- The value in a dead temporary is not needed for the rest of the computation
  - A dead temporary can be reused
- Basic rule:
  - *Temporaries t_1 and t_2 can share the same register if at any point in the program at most one of t_1 or t_2 is live !*

Algorithm: Part I

- Compute live variables for each point:

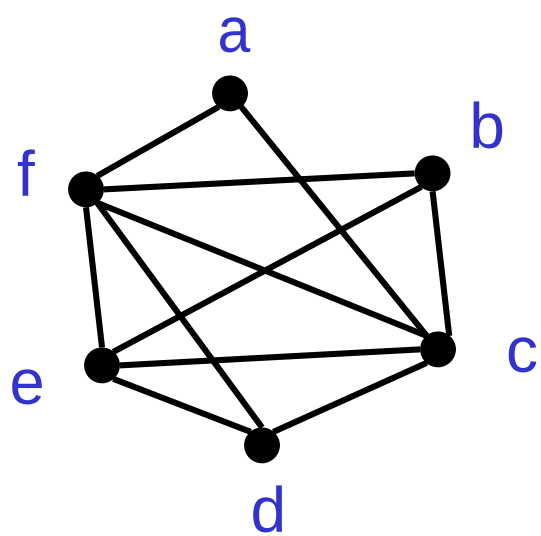


The Register Interference Graph

- Two temporaries that are live simultaneously cannot be allocated in the same register
- We construct an undirected graph
  - A node for each temporary
  - An edge between t_1 and t_2 if they are live simultaneously at some point in the program
- This is the register interference graph (RIG)
  - Two temporaries can be allocated to the same register if there is no edge connecting them

Register Interference Graph. Example.

- For our example:



- E.g., b and c cannot be in the same register
- E.g., b and d can be in the same register

Register Interference Graph. Properties.

- It extracts exactly the information needed to characterize legal register assignments
- It gives a global (i.e., over the entire flow graph) picture of the register requirements
- After RIG construction the register allocation algorithm is architecture independent

Graph Coloring. Definitions.

- A coloring of a graph is an assignment of colors to nodes, such that nodes connected by an edge have different colors

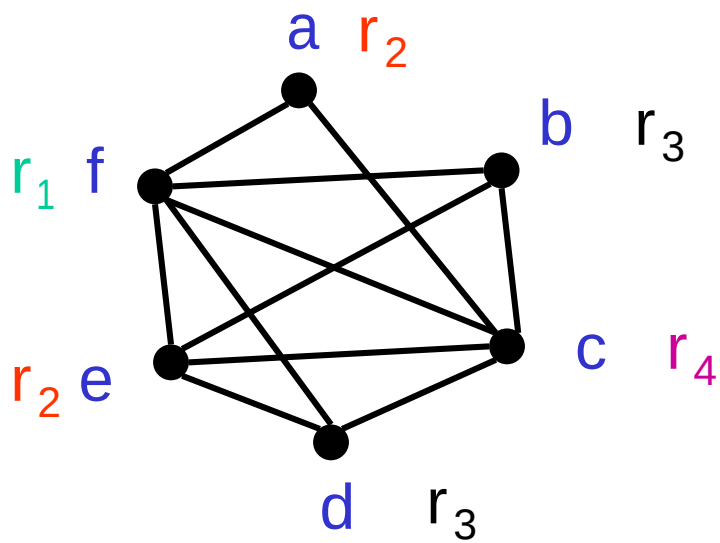
- A graph is k-colorable if it has a coloring with k colors

Register Allocation Through Graph Coloring

- In our problem, colors = registers
  - We need to assign colors (registers) to graph nodes (temporaries)
- Let k = number of machine registers
- If the RIG is k-colorable then there is a register assignment that uses no more than k registers

Graph Coloring. Example.

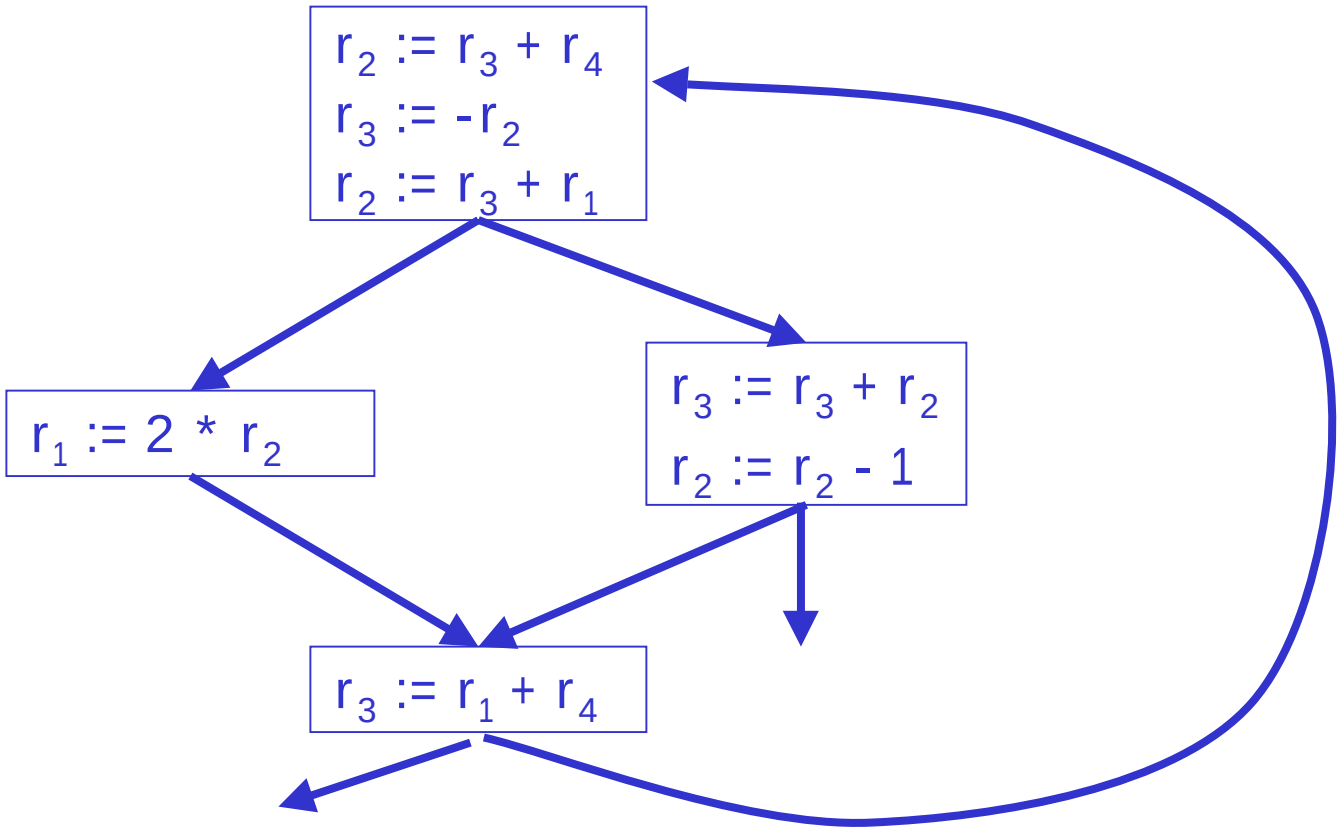
- Consider the example RIG



- There is no coloring with less than 4 colors
- There are 4-colorings of this graph

Graph Coloring. Example.

- Under this coloring the code becomes:



Computing Graph Colorings

- The remaining problem is to compute a coloring for the interference graph
- But:
  - This problem is very hard (NP-hard). No efficient algorithms are known.
  - A coloring might not exist for a given number or registers
- The solution to (1) is to use heuristics
- We'll consider later the other problem

Graph Coloring Heuristic

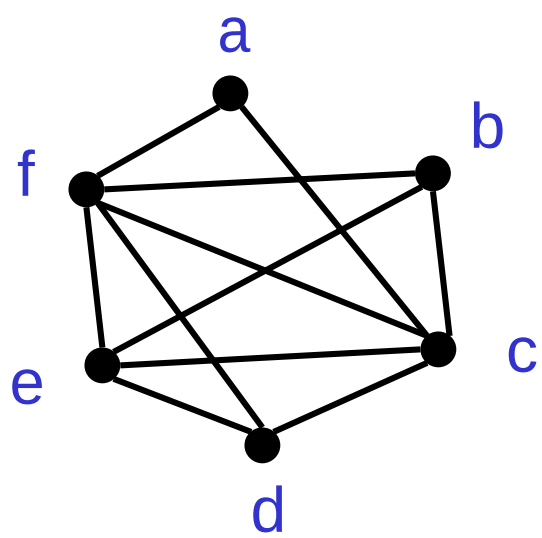
- Observation:
  - Pick a node t with fewer than k neighbors in RIG
  - Eliminate t and its edges from RIG
  - If the resulting graph has a k-coloring then so does the original graph
- Why:
  - Let $c_1,\ldots,c_n$ be the colors assigned to the neighbors of t in the reduced graph
  - Since $n < k$ we can pick some color for t that is different from those of its neighbors

Graph Coloring Heuristic

- The following works well in practice:
  - Pick a node t with fewer than k neighbors
  - Put t on a stack and remove it from the RIG
  - Repeat until the graph has one node
- Then start assigning colors to nodes on the stack (starting with the last node added)
  - At each step pick a color different from those assigned to already colored neighbors

Graph Coloring Example (1)

- Start with the RIG and with k = 4:

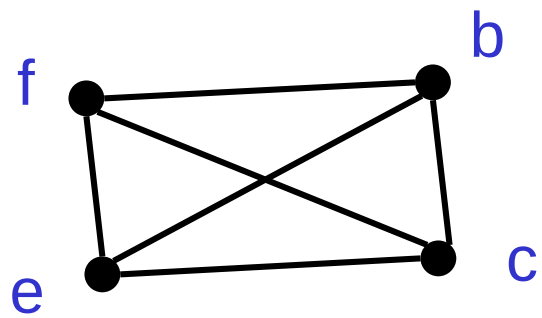


Stack: {}

- Remove a and then d

Graph Coloring Example (2)

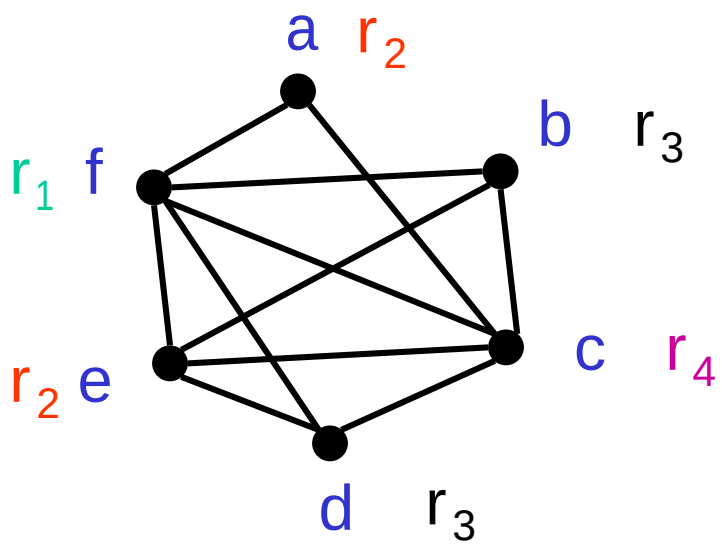
- Now all nodes have fewer than 4 neighbors and can be removed: c, b, e, f



Stack: {d, a}

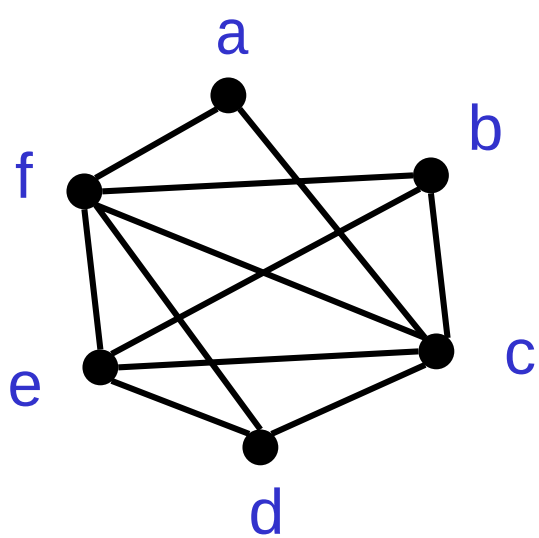
Graph Coloring Example (2)

- Start assigning colors to: f, e, b, c, d, a



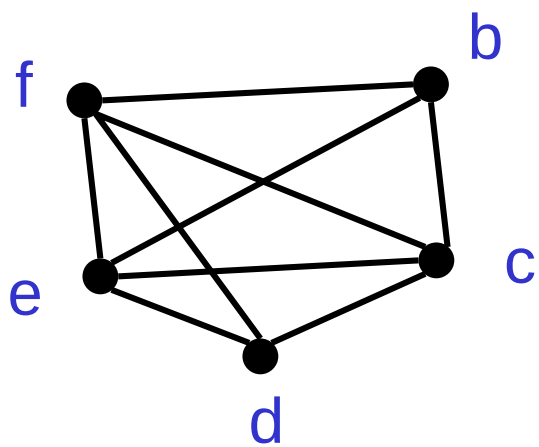
What if the Heuristic Fails?

- What if during simplification we get to a state where all nodes have k or more neighbors ?
- Example: try to find a 3-coloring of the RIG:



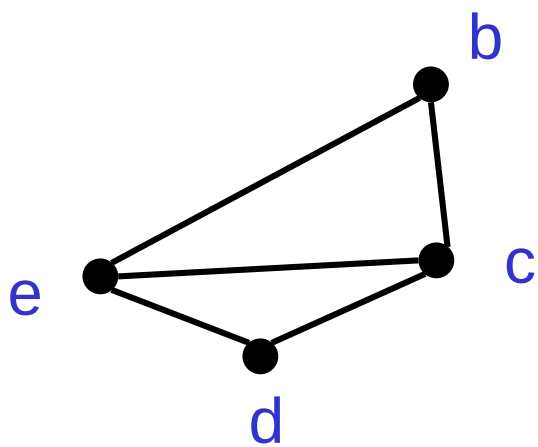
What if the Heuristic Fails?

- Remove a and get stuck (as shown below)
- Pick a node as a candidate for spilling
  - A spilled temporary "lives" in memory
- Assume that f is picked as a candidate



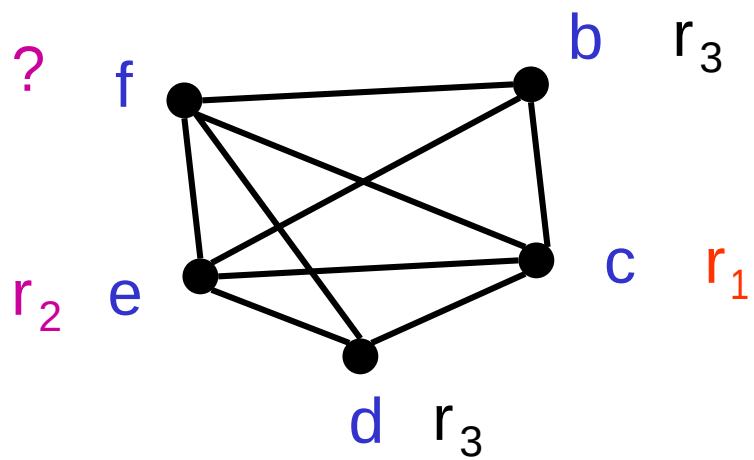
What if the Heuristic Fails?

- Remove f and continue the simplification
  - Simplification now succeeds: b, d, e, c



What if the Heuristic Fails?

- On the assignment phase we get to the point when we have to assign a color to f
- We hope that among the 4 neighbors of f we use less than 3 colors ⇒ optimistic coloring



Spilling

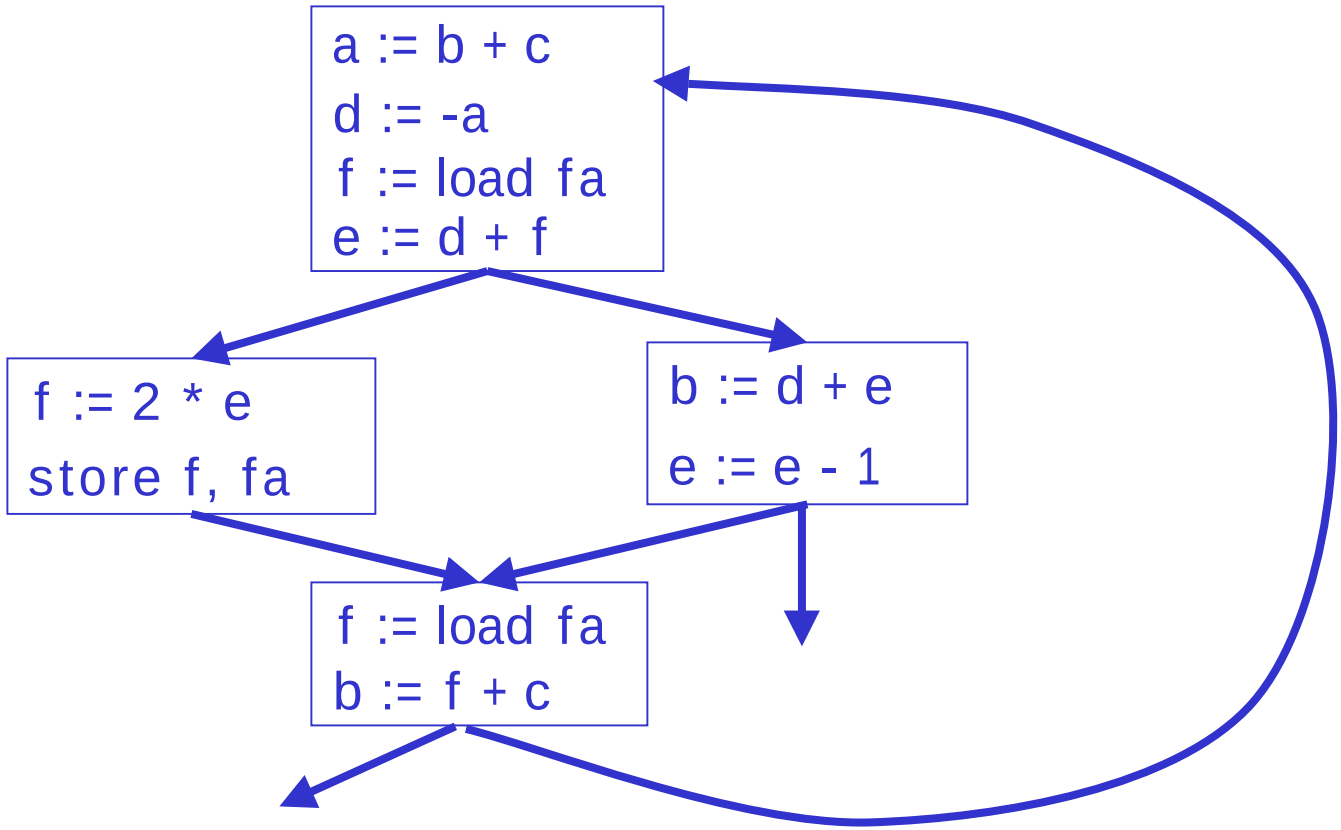
- Since optimistic coloring failed we must spill temporary f
- We must allocate a memory location as the home of f
  - Typically this is in the current stack frame
  - Call this address fa
- Before each operation that uses f, insert

  f := load fa
- After each operation that defines f, insert

  store f, fa

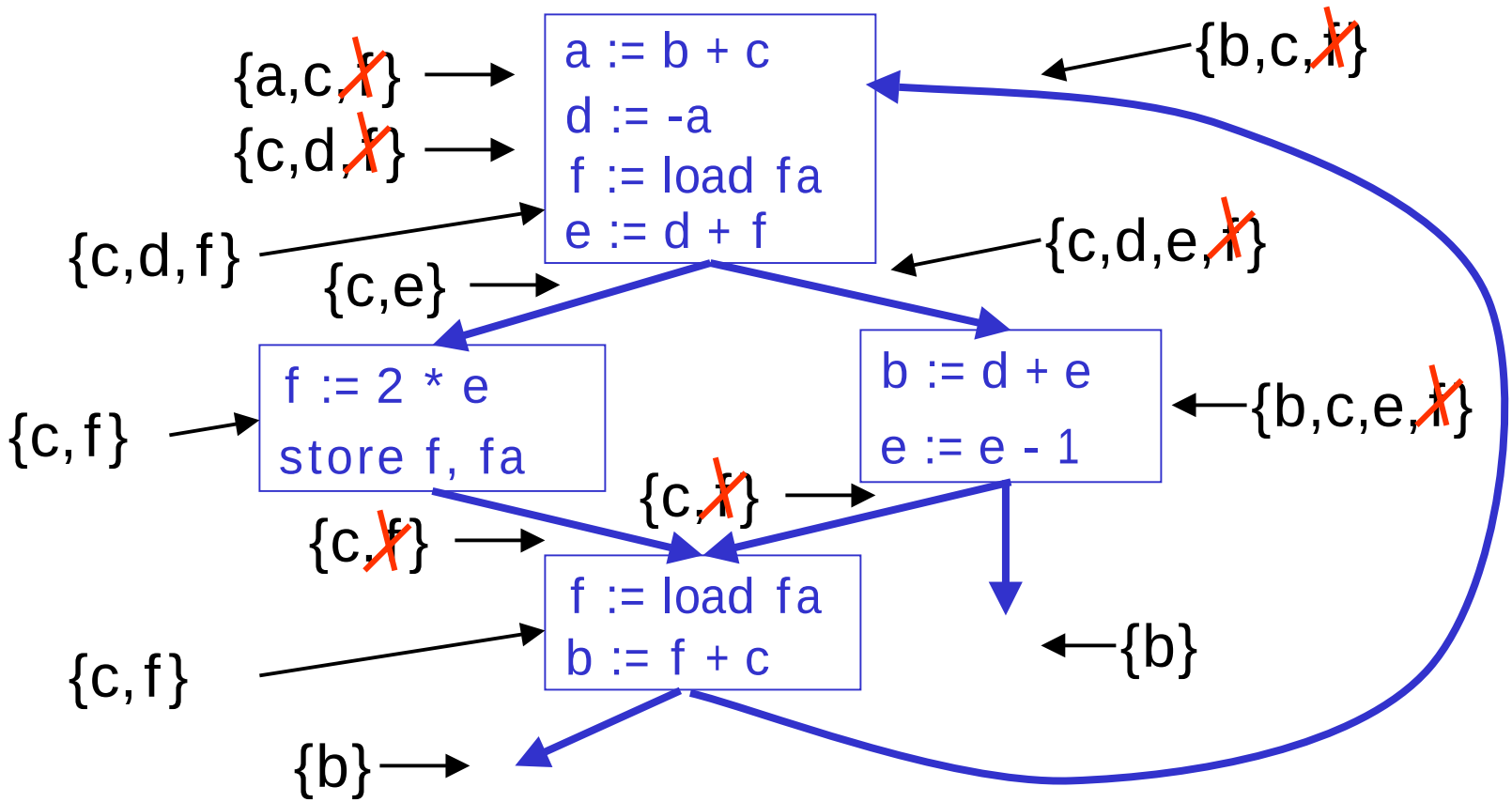
Spilling. Example.

- This is the new code after spilling f



Recomputing Liveness Information

- The new liveness information after spilling:

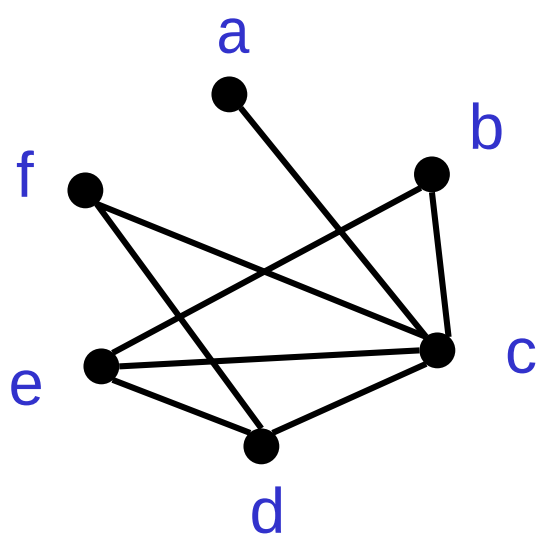


Recomputing Liveness Information

- The new liveness information is almost as before
- f is live only
  - Between a f := load fa and the next instruction
  - Between a store f, fa and the preceding instr.
- Spilling reduces the live range of f
- And thus reduces its interferences
- Which result in fewer neighbors in RIG for f

Recompute RIG After Spilling

- The only changes are in removing some of the edges of the spilled node
- In our case f still interferes only with c and d
- And the resulting RIG is 3-colorable



Spilling (Cont.)

- Additional spills might be required before a coloring is found
- The tricky part is deciding what to spill
- Possible heuristics:
  - Spill temporaries with most conflicts
  - Spill temporaries with few definitions and uses
  - Avoid spilling in inner loops
- Any heuristic is correct

Caches

- Compilers are very good at managing registers
  - Much better than a programmer could be
- Compilers are not good at managing caches
  - This problem is still left to programmers
  - It is still an open question whether a compiler can do anything general to improve performance
- Compilers can, and a few do, perform some simple cache optimization

Cache Optimization

- Consider the loop

```
for(j := 1; j < 10; j++)
    for(i=1; i<1000; i++)
        a[i] *= b[i]
```

  - This program has a terrible cache performance
    - Why?

Cache Optimization (Cont.)

- Consider the program:

```
for(i=1; i<1000; i++)
    for(j := 1; j < 10; j++)
        a[i] *= b[i]
```

  - Computes the same thing
  - But with much better cache behavior
  - Might actually be more than 10x faster
- A compiler can perform this optimization
  - called loop interchange

Conclusions

- Register allocation is a “must have” optimization in most compilers:
  - Because intermediate code uses too many temporaries
  - Because it makes a big difference in performance
- Graph coloring is a powerful register allocation schemes
- Register allocation is more complicated for CISC machines