Symmetric Functions

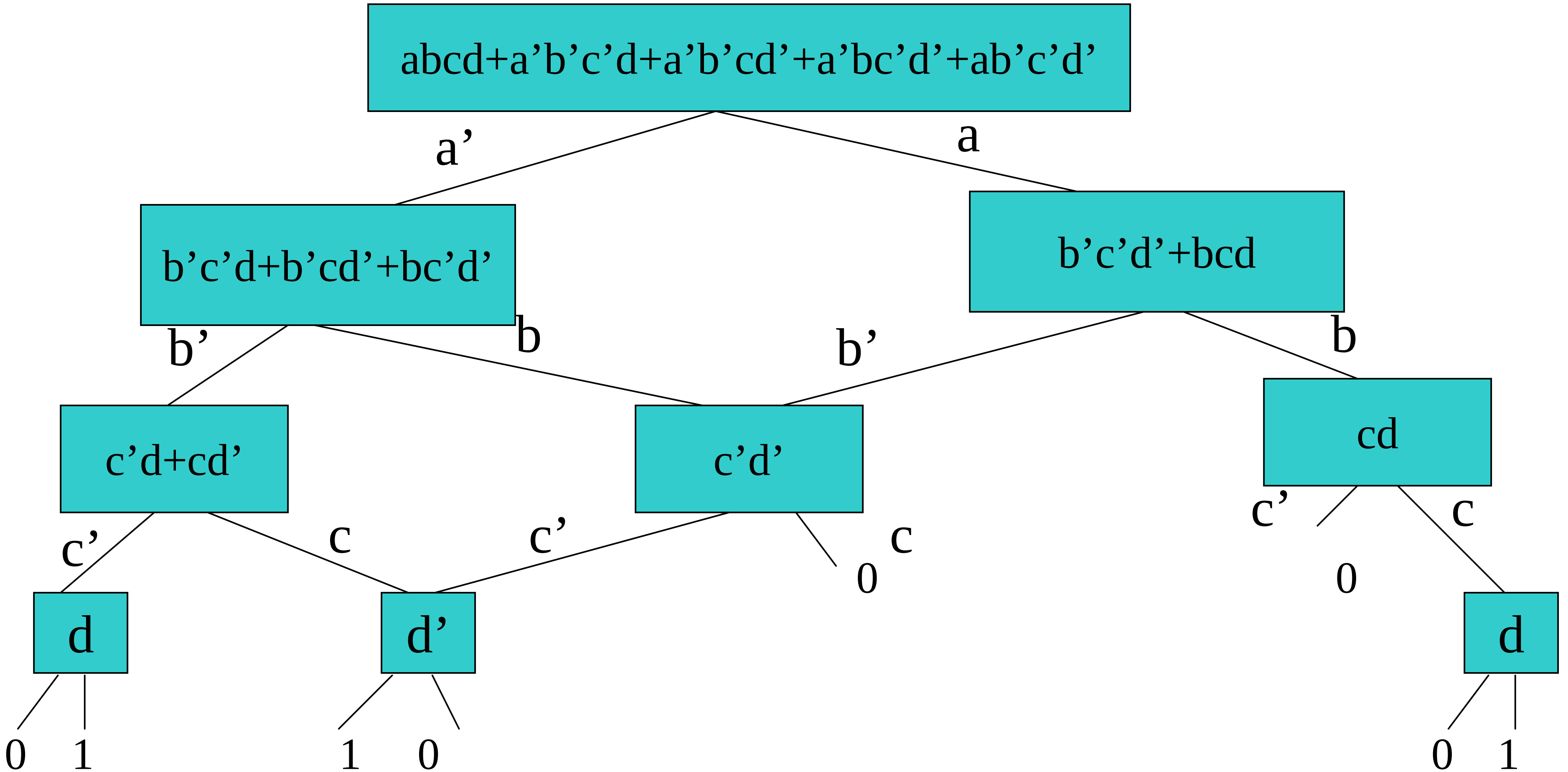
Symmetric functions and their representation. Their applications. Examples. Layouts, circuits,notation,synthesis methods.

Symmetric Functions

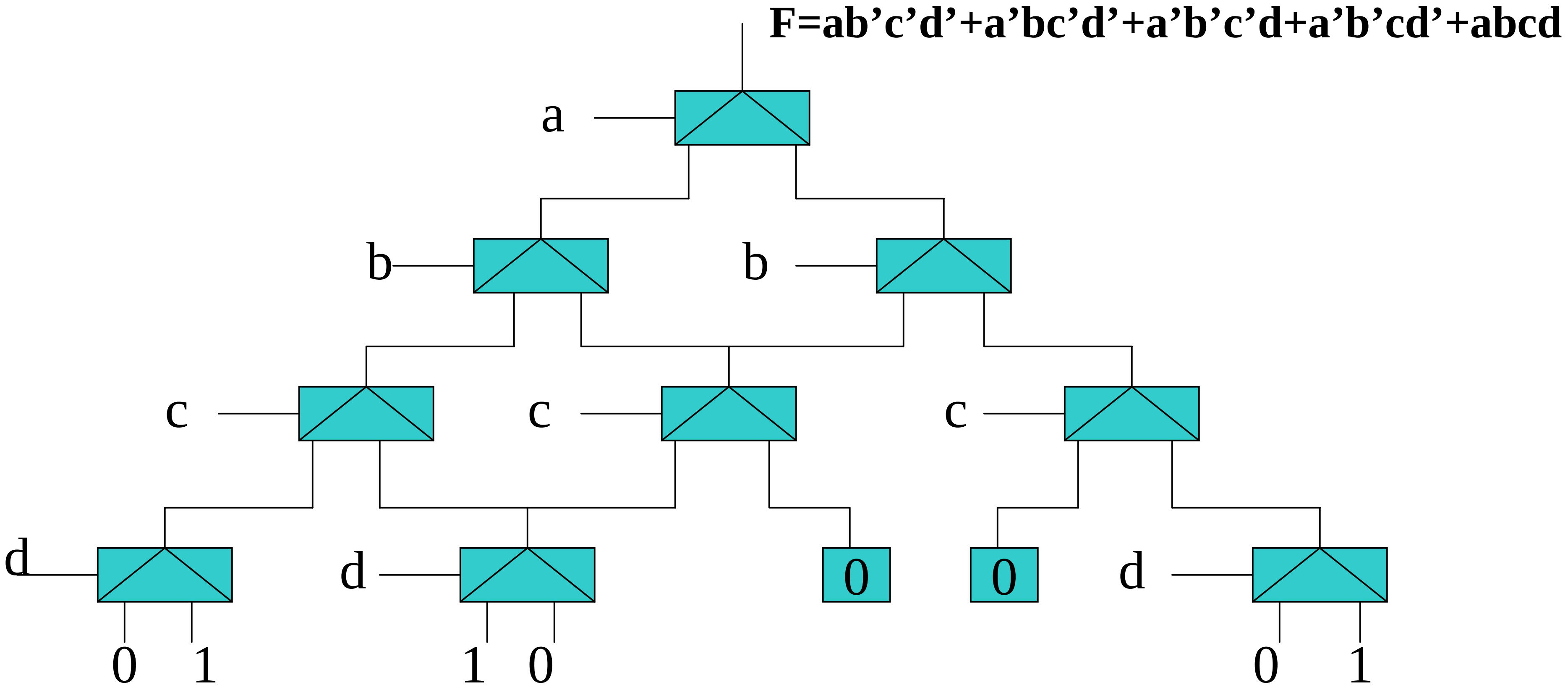
- Important topic
- Review various definitions of symmetry.

SHANNON EXPANSION OF THE FUNCTION

This function is symmetric!

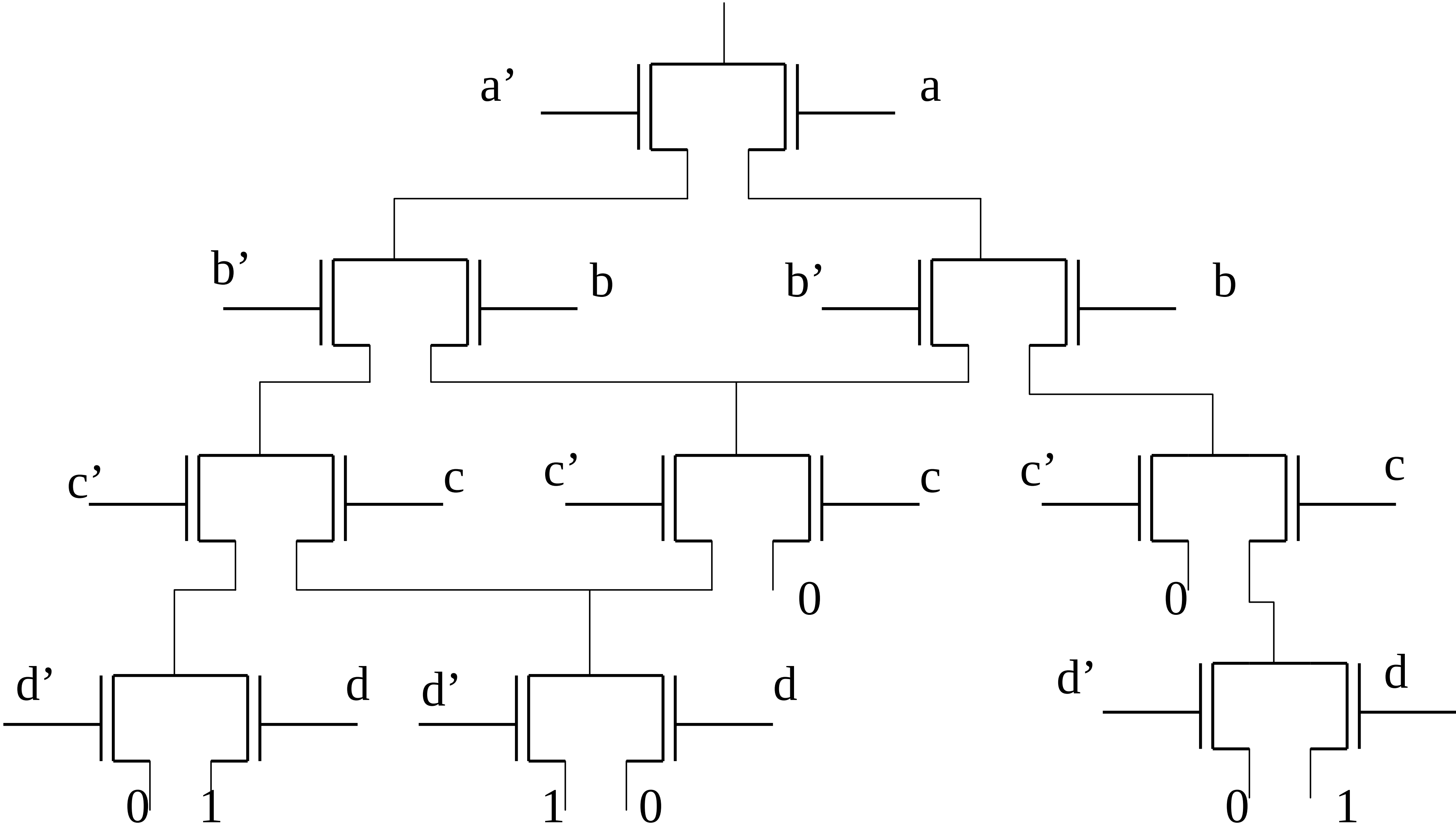


MULTIPLEXOR CIRCUIT REALIZATION OF FUNCTION

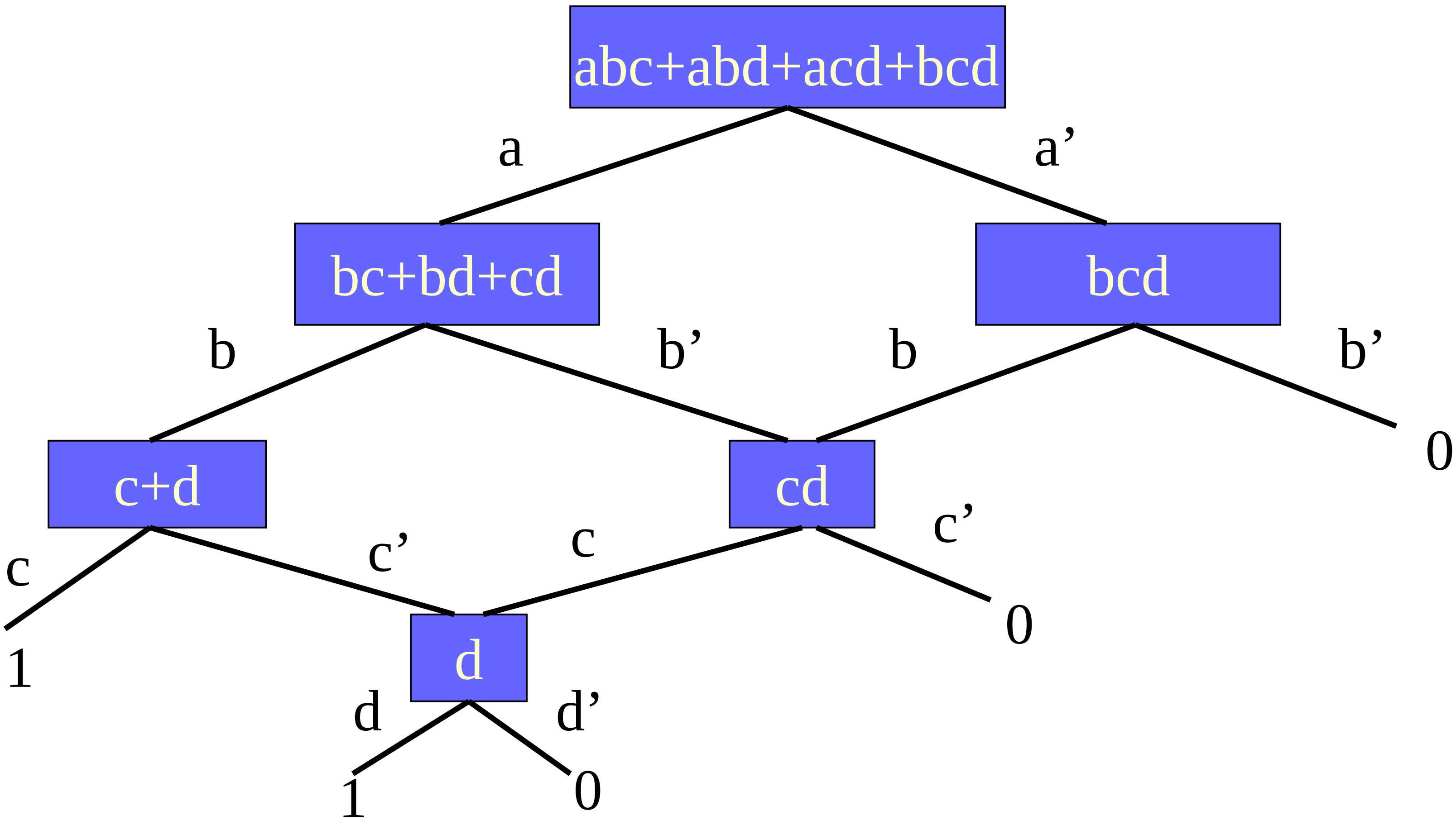


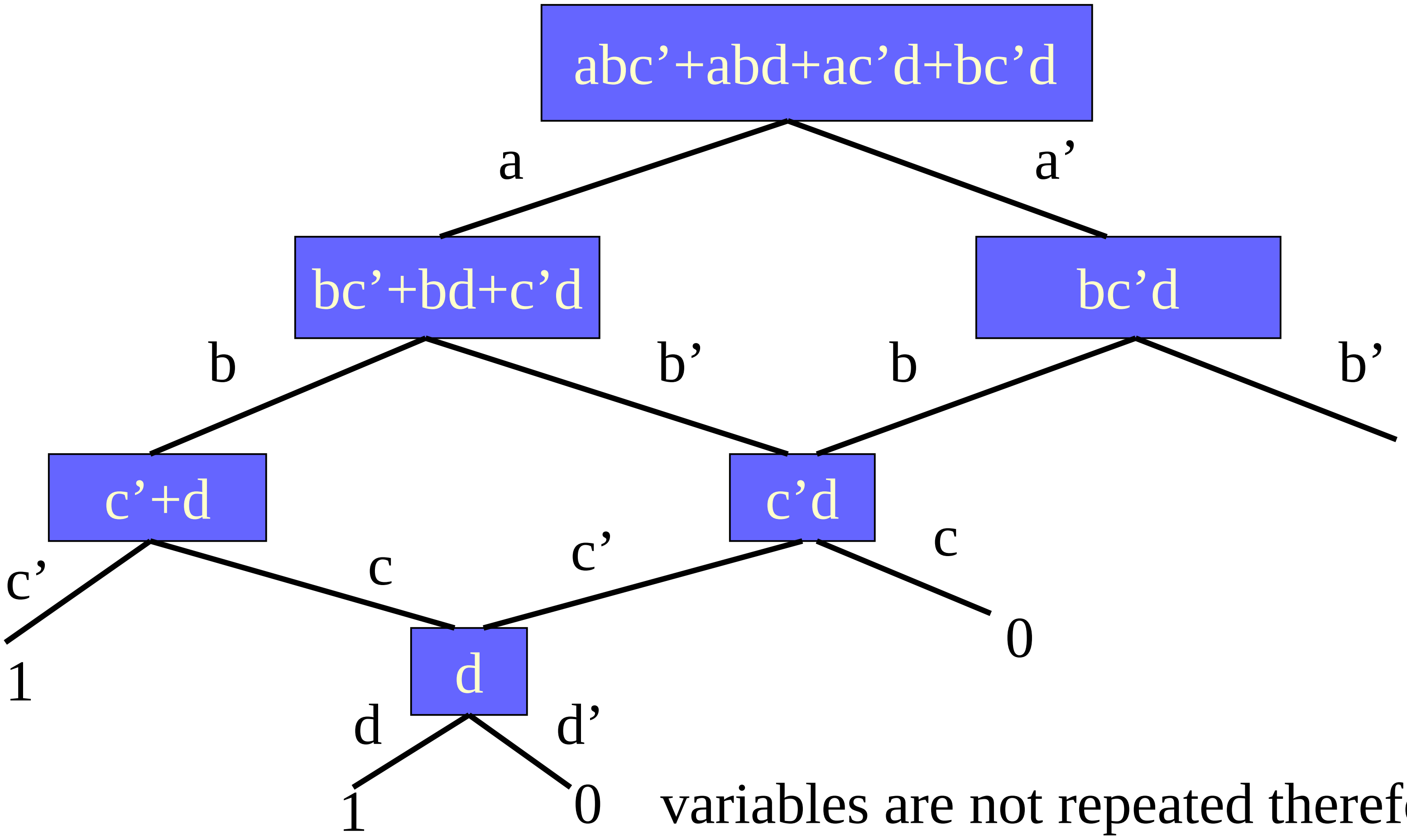
PASS TRANSISTOR LAYOUT OF A FUNCTION

F=a’b’c’d+ab’c’d’+a’bc’d’+a’b’cd’+abcd



BDDs of functions shown in the previous slide





variables are not repeated therefore, both are symmetric functions.

DEFINITON AND PROPERTIES OF SYMMETRIC FUNCTIONS

Definitions:

A switching function of n variables $f(x_1,x_2,\ldots\ldots..x_n)$ is called ***symmetric*** or ***totally symmetric*** if and only if it is invariant under any permutation of variables ;

It is called ***partially symmetric*** in the variables x_i,x_j, where $\{x_i,x_j\}$ is a subset of $\{x_1,x_2,\ldots\ldots..x_n\}$, if and only if the interchange of the variables x_i,x_j leaves the function unchanged.

Example: f(a,b,c)=a’b’c+ab’c’+a’bc’ is symmetric because interchange between variables doesn’t change the function.where as f(a,b,c) = a’b’c+ab’c’ is partially symmetric in the variables a and c.

•The variable in which function is symmetric is called variable of symmetry.A symmetric function is denoted $S^{a_1,a_2\ldots ak}(x_1,x_2,\ldots\ldots\ldots x_n)$,
where S designates the property of symmetry, the superscripts a1,…ak designate the a-numbers, and $(x_1,x_2,\ldots\ldots\ldots x_n)$ designates the variables of symmetry.

Example-1: The function f(a,b,c)=a’b’c+a’bc’+ab’c’ assumes the value ‘1’when and only when one out of its three variables is ‘1’.

This function is denoted as $S^1(a,b,c)$, similarly the symmetric function $S^{1,3}(a,b,c)$ is f(a,b,c)=abc+a’b’c+ab’c’+a’bc’

Definition-2: Let f1(a,b,c,d) = $\mathbf{S}_{0,2,4}$**(a,b,c,d)** and f2(a,b,c,d) = $\mathbf{S}_{3,4}$**(a,b,c,d)**

then f3(a,b,c,d) = f1+f2 = $\mathbf{S}_{0,2,3,4}$**(a,b,c,d)**

and f4(a,b,c,d) = f1*f2 = $\mathbf{S}_{4}$**(a,b,c,d).**

•The **complement** of the symmetric function is also a symmetric function whose a-numbers are included in the set {0,1..n}and not included in the original function.

for example $\mathbf{S'}_{0,2,4}$**(a,b,c,d)** = $\mathbf{S}_{1,3}$**(a,b,c,d)** for set of {0,1,2,3,4}.

REPRESENTATION OF SYMMETRIC FUNCTIONS

•The basic network for symmetric function is shown in the next slide network is drown for four variables it can be extended for n variables. It is a multi output network consisting of a single input and 5output numbered from 0 to 4 .

• Network which realizes symmetric function is called symmetric network.Contacts of symmetric network are arranged in such a way that input can propagate in two directions.

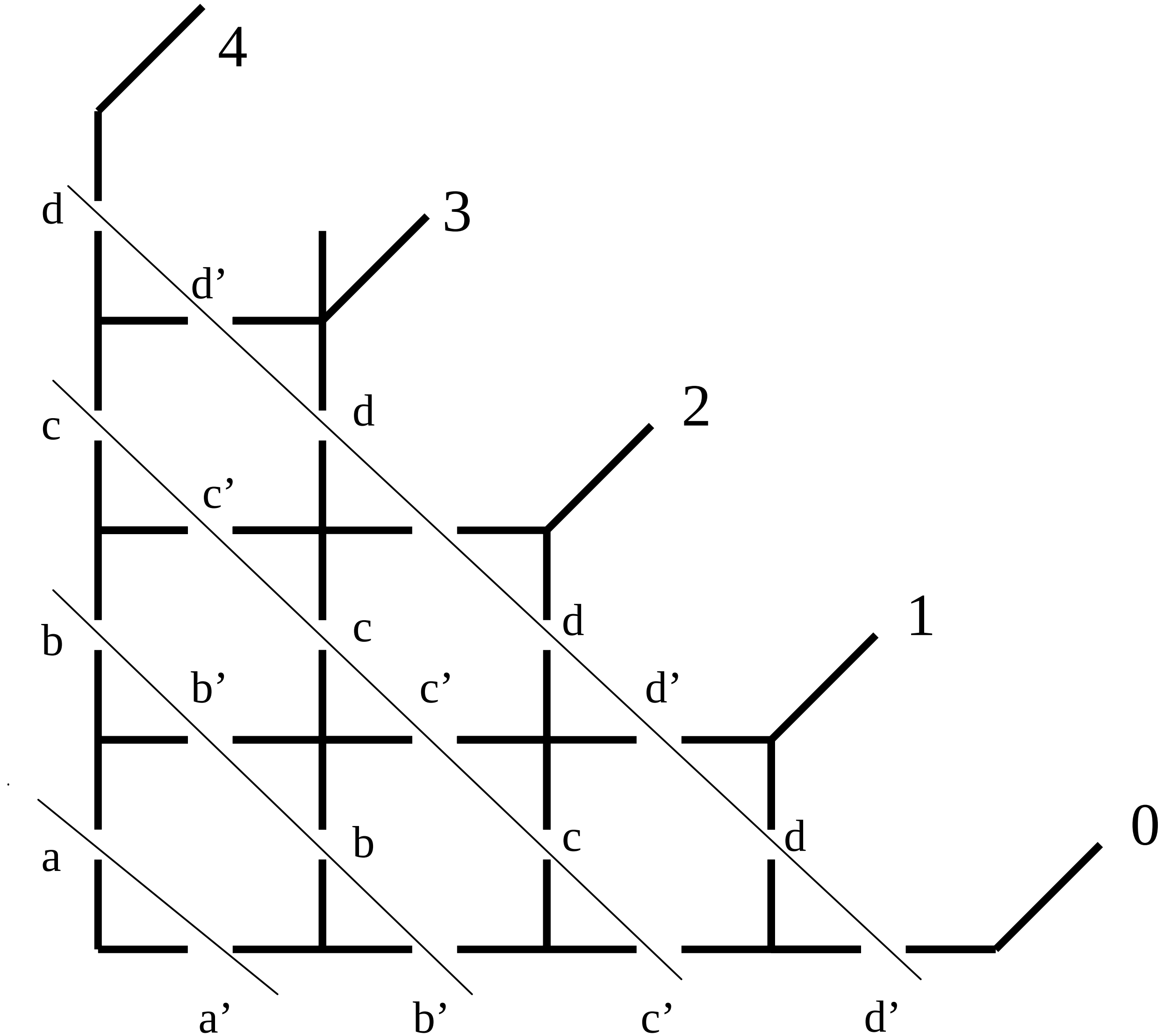
-From bottom to top

-And from left to right

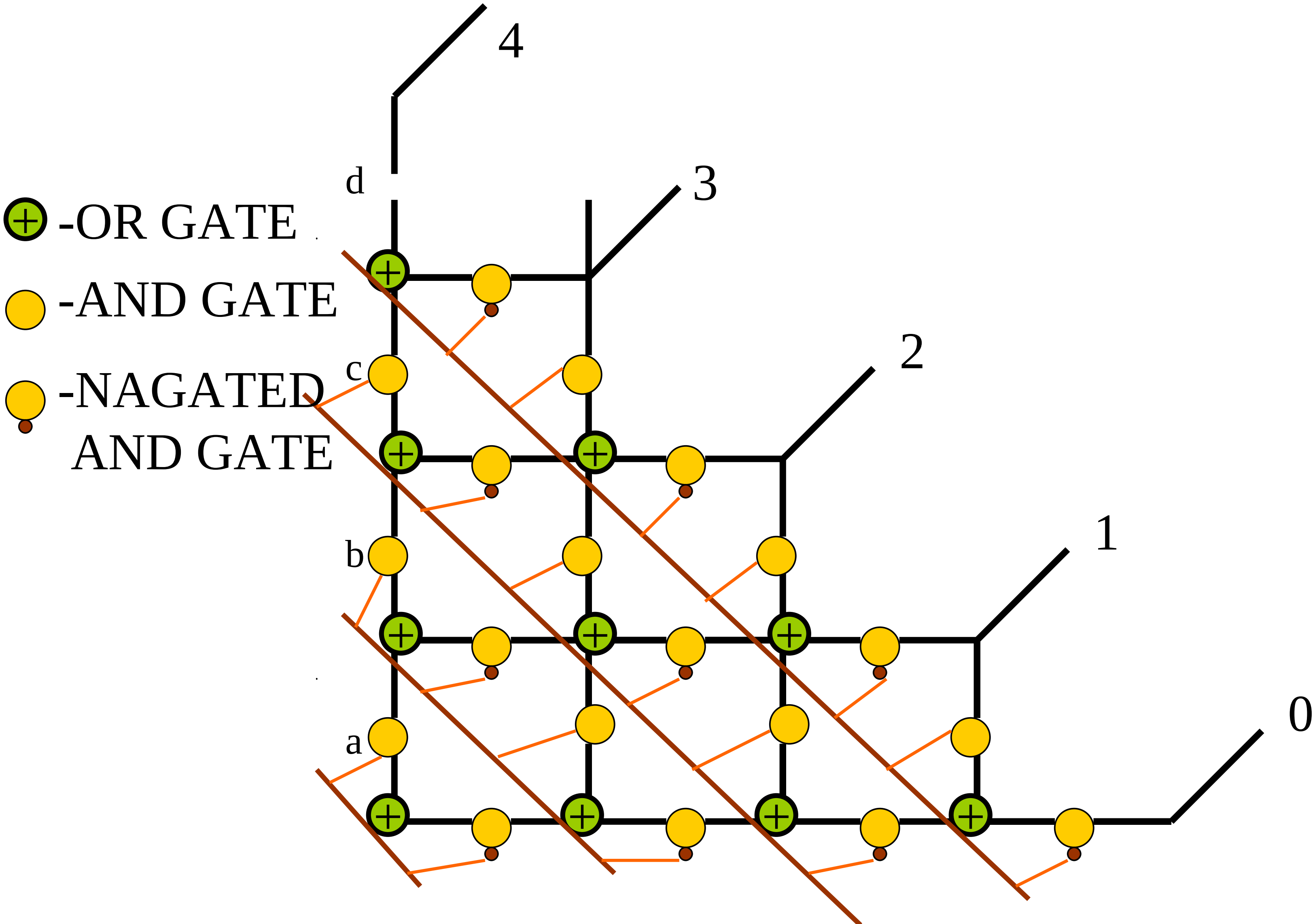
•Contacts of the operated relays shifts input upward to successive level, while contacts of the un-operated relays shifts input to the right.

These properties of the symmetric function make it possible to simplify the network in the various ways.

SWITCH REALIZATION OF THE SYMMETRIC FUNCTION



LATTICE REALIZATION OF SYMMETRIC FUNCTION



LATTICE RELIZATION OF SYMMETRIC FUNCTION

•Previous slide showed multiplexor sub circuit drawn on the switch realization.

•Each multiplexor has a (input) control variable ,and two data input selected by a state 0 or 1 of the control variable.

•For variable b the inputs 0 and 1 correspond to b’ and b respectively,

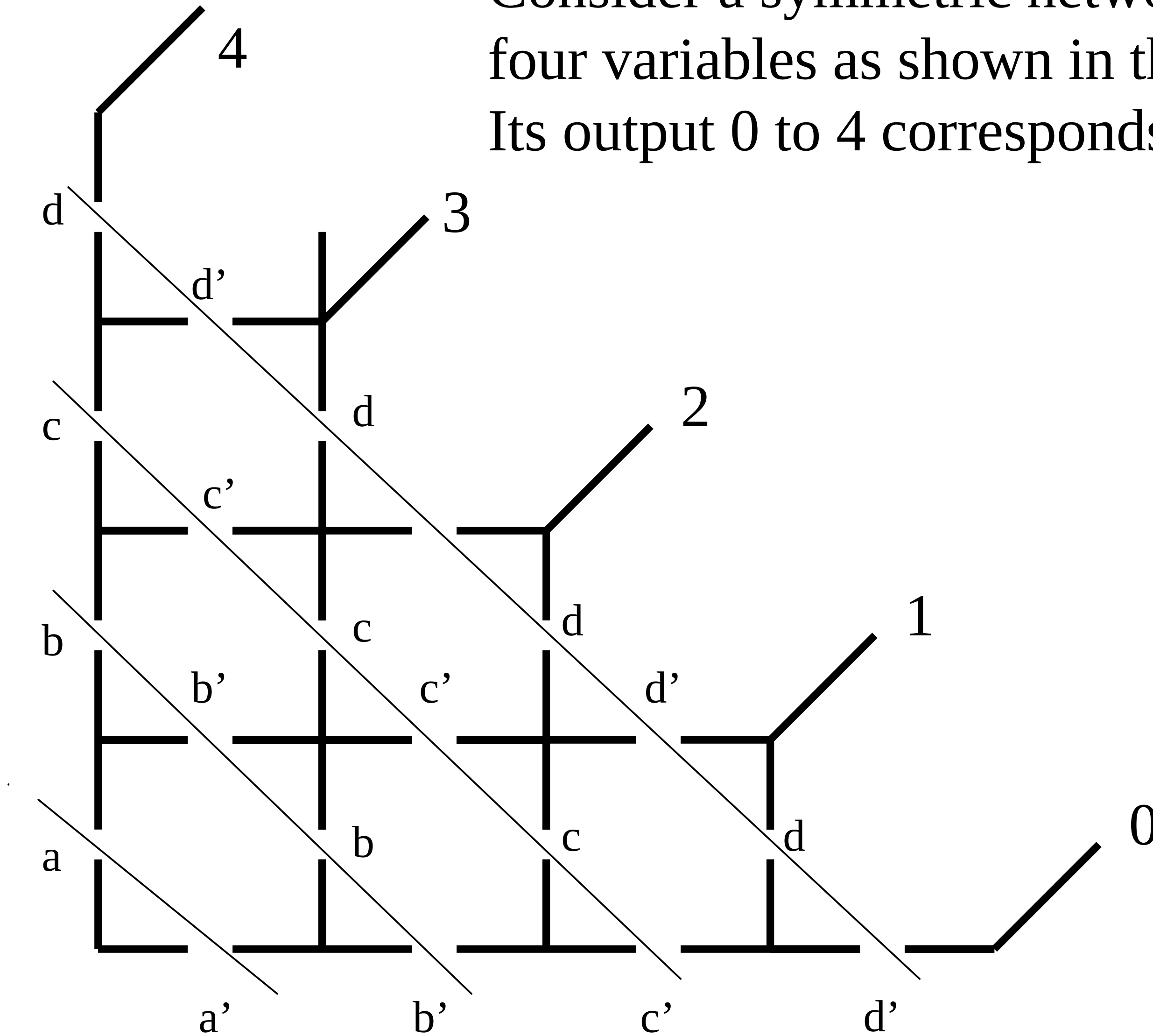
• This circuit directly leads to layout.

• Observe that every variable from the diagonal bus goes to negated input of the and gate and non-negated input of the and gate of the multiplexor with or gate as its output.

- Each output corresponding to symmetry of function is set of a Boolean constant.
- Each multiplexor in the lattice obtains one data input from north and one from east, and directs its output to south and west.
- Diagonal lines are input control variable of multiplexors.

EXAMPLE OF SYMMETRIC NETWORK, FUNCTION REALIZATION AND SYNTHESIS

Consider a symmetric network of four variables as shown in the figure Its output 0 to 4 corresponds to a no.



Let us realize symmetric function S1,4(a,b,c,d)

-It is necessary to join output terminals labeled 1 and 4 as shown in the next slide.

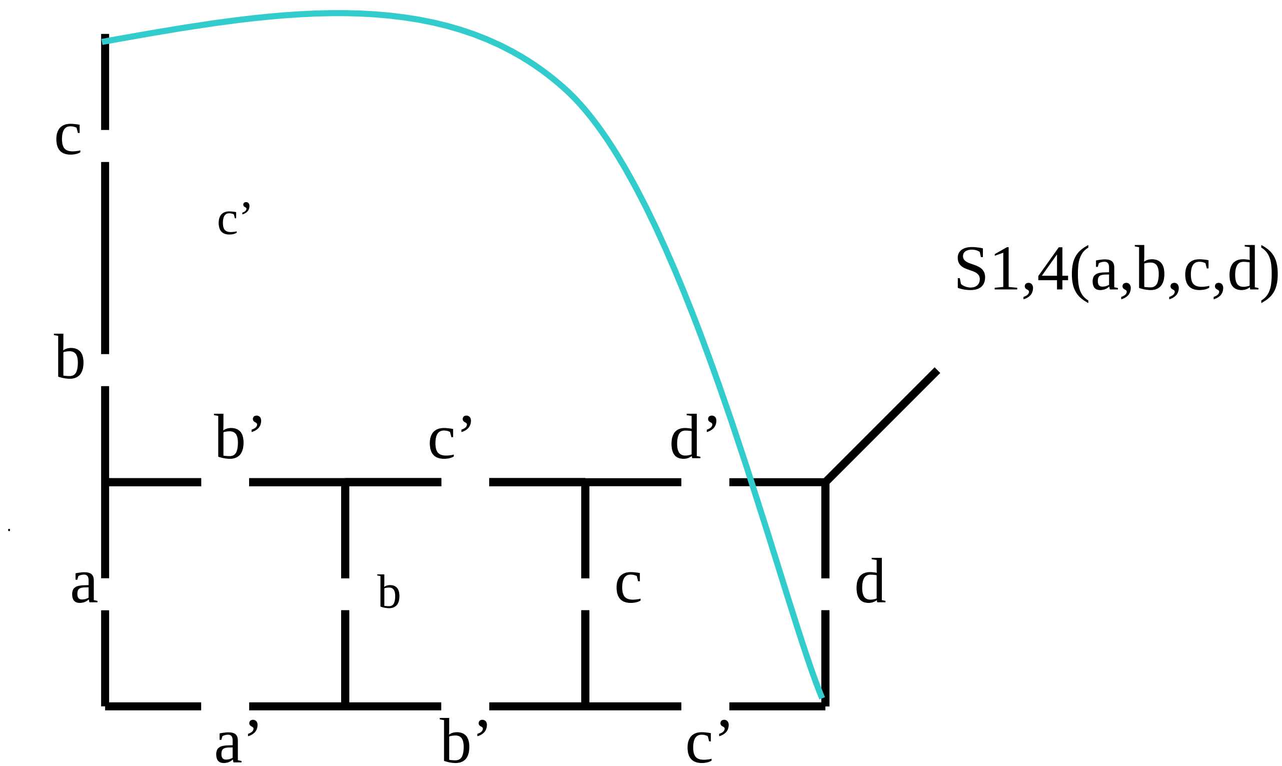
-It is required to delete all unused terminal as shown in the next slide.

-Minimal network of the function can be achieved as shown in the slide

-This simplified network represents the Boolean function $f=a'b'c'd+a'b'cd'+a'bc'd'+ab'c'd'+abcd$.

This simplification of symmetric network is called synthesis of symmetric network.

SYNTHESIS OF SYMMETRIC NETWORK



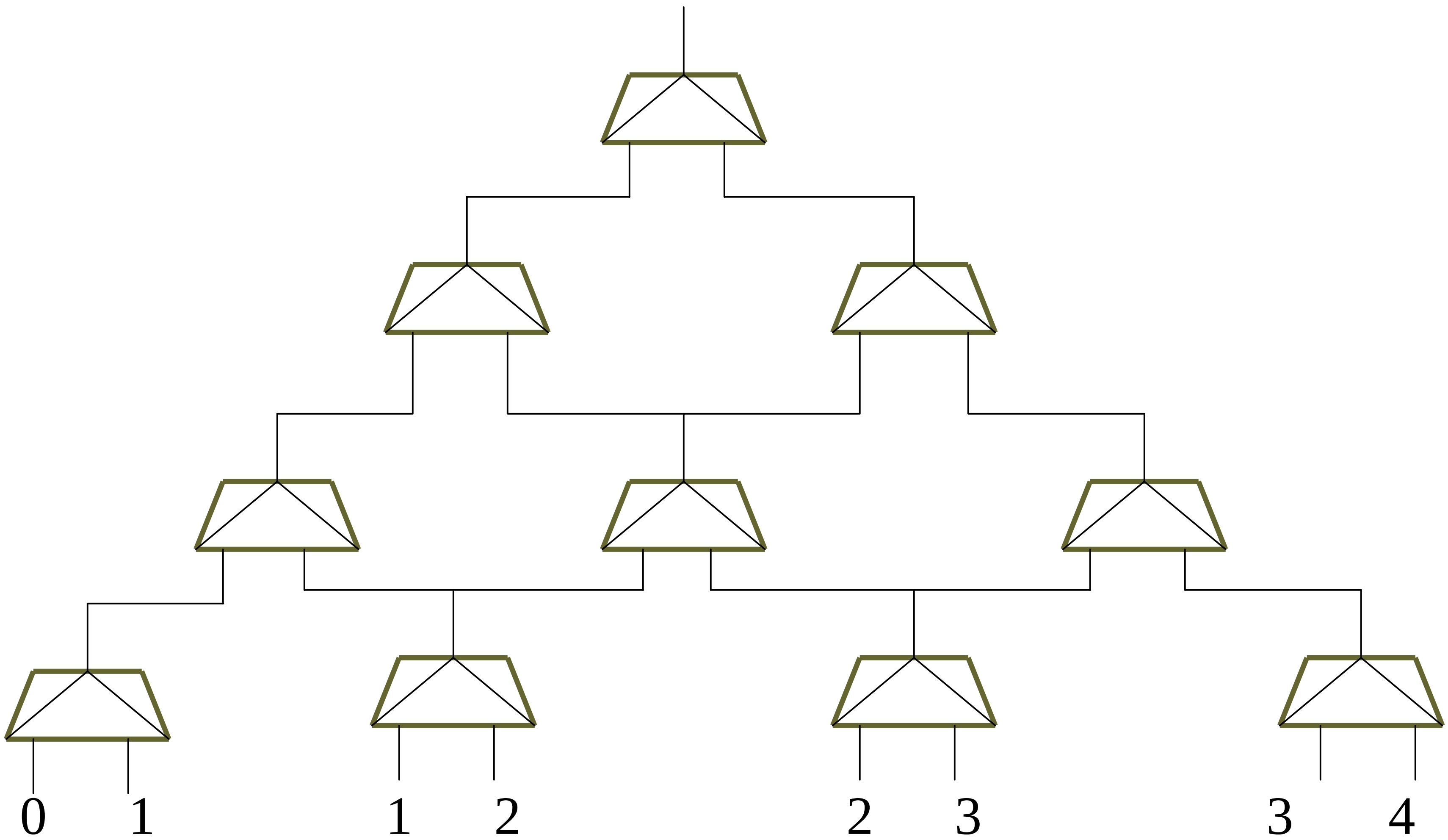
APPLICATION

Symmetry in FPGA layout- Consider the K'map of 4 variables shown below,This is fully symmetric function . It has integer value in each of the boxes.Here the Boolean outputs in the square labeled 1 are equal , labeled 2 are equal,labeled 3 are equal and labeled 4 are equal.

	00	01	11	10
00	0	1	2	1
01	1	2	3	2
11	2	3	4	3
10	1	2	3	2

Other applications of symmetric functions are in arithmetic circuits such as full adder circuit

This function can be mapped in to the lattice of multiplexer as shown in the figure below.



OBSERVATION

Mirror image function of any symmetric function is symmetric for same negated variable for which original function is symmetric.

For example :

ab \ cd	00	01	11	10
00				
01			1	
11		1	1	1
10			1	

F=abc+abd+acd+bcd

=

ab \ cd	00	01	11	10
00				
01		1		
11	1	1	1	
10		1		

F=abc’+abd+c’db+c’da

both are symmetric function

Problems to think about

- Symmetry versus BDDs, KFDDs, ZBDDs and other DAG-based representations
- Regular structures for multi-output functions, based on symmetries or partial symmetries
- Finding partial symmetry in non-symmetrical function
- How to generalize the concepts of symmetry?
- Symmetry and pass-transistor logic
- Symmetry of Multi-Valued functions